AQA Maths FP1

Past Paper Pack

2006-2014

General Certificate of Education
January 2006
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Monday 23 January 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Question 6 (enclosed)

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 (a) Show that the equation

$$x^3 + 2x - 2 = 0$$

has a root between 0.5 and 1. *(2 marks)*

(b) Use linear interpolation once to find an estimate of this root. Give your answer to two decimal places. *(3 marks)*

2 (a) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(i) $\displaystyle\int_0^9 \frac{1}{\sqrt{x}}\,\mathrm{d}x$; *(3 marks)*

(ii) $\displaystyle\int_0^9 \frac{1}{x\sqrt{x}}\,\mathrm{d}x$. *(3 marks)*

(b) Explain briefly why the integrals in part (a) are improper integrals. *(1 mark)*

3 Find the general solution, in **degrees**, for the equation

$$\sin(4x + 10°) = \sin 50°$$ *(5 marks)*

4 A curve has equation

$$y = \frac{6x}{x-1}$$

(a) Write down the equations of the two asymptotes to the curve. *(2 marks)*

(b) Sketch the curve and the two asymptotes. *(4 marks)*

(c) Solve the inequality

$$\frac{6x}{x-1} < 3$$ *(4 marks)*

5 (a) (i) Calculate $(2 + i\sqrt{5})(\sqrt{5} - i)$. *(3 marks)*

(ii) Hence verify that $\sqrt{5} - i$ is a root of the equation

$$(2 + i\sqrt{5})\, z = 3z^*$$

where z^* is the conjugate of z. *(2 marks)*

(b) The quadratic equation

$$x^2 + px + q = 0$$

in which the coefficients p and q are real, has a complex root $\sqrt{5} - i$.

(i) Write down the other root of the equation. *(1 mark)*

(ii) Find the sum and product of the two roots of the equation. *(3 marks)*

(iii) Hence state the values of p and q. *(2 marks)*

6 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = kx^n$$

where k and n are constants.

Experimental evidence has provided the following approximate values:

x	4	17	150	300
y	1.8	5.0	30	50

(a) Complete the table in **Figure 1**, showing values of X and Y, where

$$X = \log_{10} x \quad \text{and} \quad Y = \log_{10} y$$

Give each value to two decimal places. *(3 marks)*

(b) Show that if $y = kx^n$, then X and Y must satisfy an equation of the form

$$Y = aX + b$$

(3 marks)

(c) Draw on **Figure 2** a linear graph relating X and Y. *(3 marks)*

(d) Find an estimate for the value of n. *(2 marks)*

Turn over for the next question

Turn over ▶

7 (a) The transformation T is defined by the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(i) Describe the transformation T geometrically. *(2 marks)*

(ii) Calculate the matrix product $\mathbf{A}^2$. *(2 marks)*

(iii) Explain briefly why the transformation T followed by T is the identity transformation. *(1 mark)*

(b) The matrix $\mathbf{B}$ is defined by

$$\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(i) Calculate $\mathbf{B}^2 - \mathbf{A}^2$. *(3 marks)*

(ii) Calculate $(\mathbf{B} + \mathbf{A})(\mathbf{B} - \mathbf{A})$. *(3 marks)*

8 A curve has equation $y^2 = 12x$.

(a) Sketch the curve. *(2 marks)*

(b) (i) The curve is translated by 2 units in the positive y direction. Write down the equation of the curve after this translation. *(2 marks)*

(ii) The **original** curve is reflected in the line $y = x$. Write down the equation of the curve after this reflection. *(1 mark)*

(c) (i) Show that if the straight line $y = x + c$, where c is a constant, intersects the curve $y^2 = 12x$, then the x-coordinates of the points of intersection satisfy the equation

$$x^2 + (2c - 12)x + c^2 = 0$$ *(3 marks)*

(ii) Hence find the value of c for which the straight line is a tangent to the curve. *(2 marks)*

(iii) Using this value of c, find the coordinates of the point where the line touches the curve. *(2 marks)*

(iv) In the case where $c = 4$, determine whether the line intersects the curve or not. *(3 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
January 2006
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Insert

Monday 23 January 2006 1.30 pm to 3.00 pm

Insert for use in **Question 6.**

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

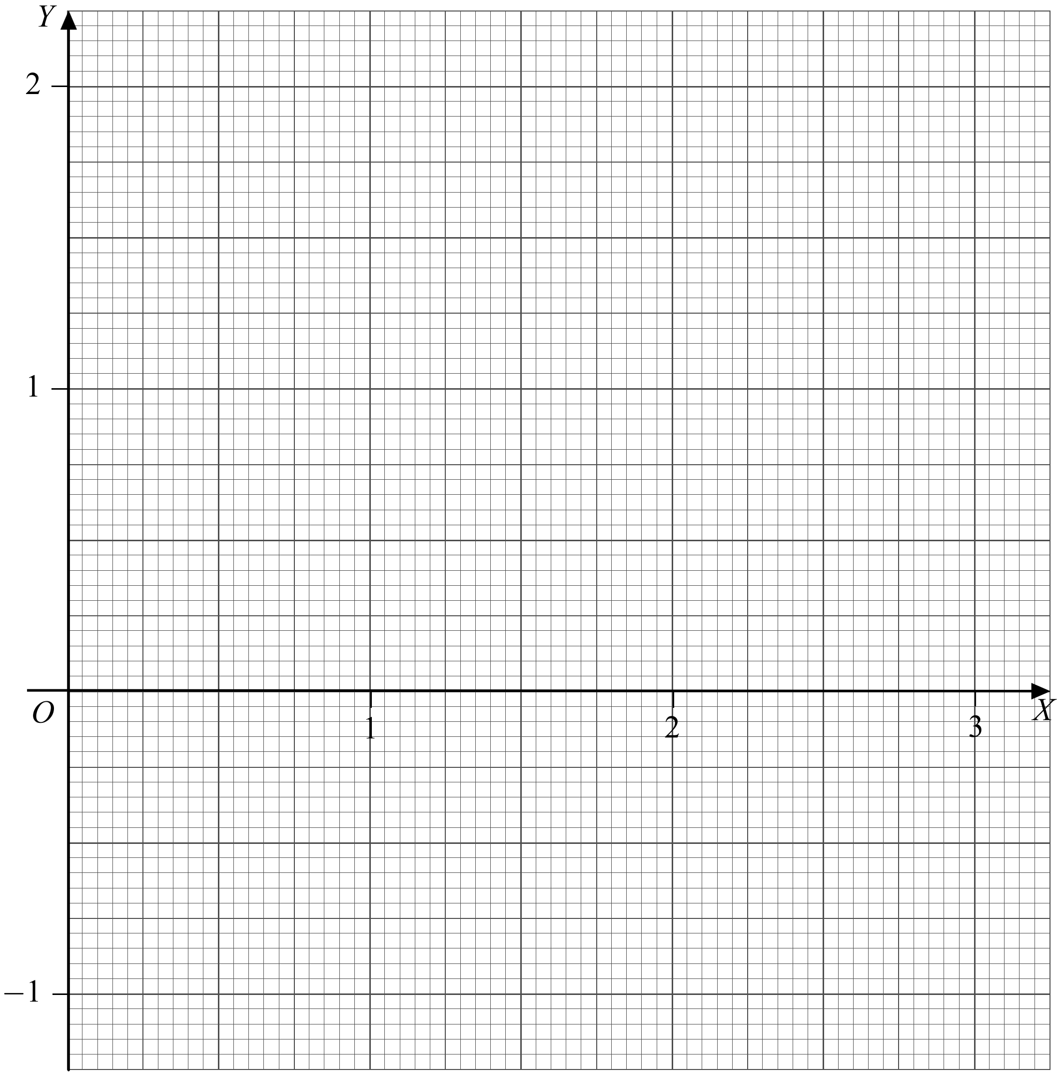
Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 6)

X	0.60			2.48
Y	0.26			1.70

Figure 2 (for use in Question 6)



General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Friday 26 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Solve the following equations, giving each root in the form $a + b\mathrm{i}$:

(i) $x^2 + 16 = 0$; *(2 marks)*

(ii) $x^2 - 2x + 17 = 0$. *(2 marks)*

(b) (i) Expand $(1 + x)^3$. *(2 marks)*

(ii) Express $(1 + \mathrm{i})^3$ in the form $a + b\mathrm{i}$. *(2 marks)*

(iii) Hence, or otherwise, verify that $x = 1 + \mathrm{i}$ satisfies the equation

$$x^3 + 2x - 4\mathrm{i} = 0$$ *(2 marks)*

2 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}$$

(a) Calculate:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $\mathbf{BA}$. *(3 marks)*

(b) Describe fully the geometrical transformation represented by each of the following matrices:

(i) $\mathbf{A}$; *(2 marks)*

(ii) $\mathbf{B}$; *(2 marks)*

(iii) $\mathbf{BA}$. *(2 marks)*

3 The quadratic equation

$$2x^2 + 4x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = 1$. *(3 marks)*

(c) Find the value of $\alpha^4 + \beta^4$. *(3 marks)*

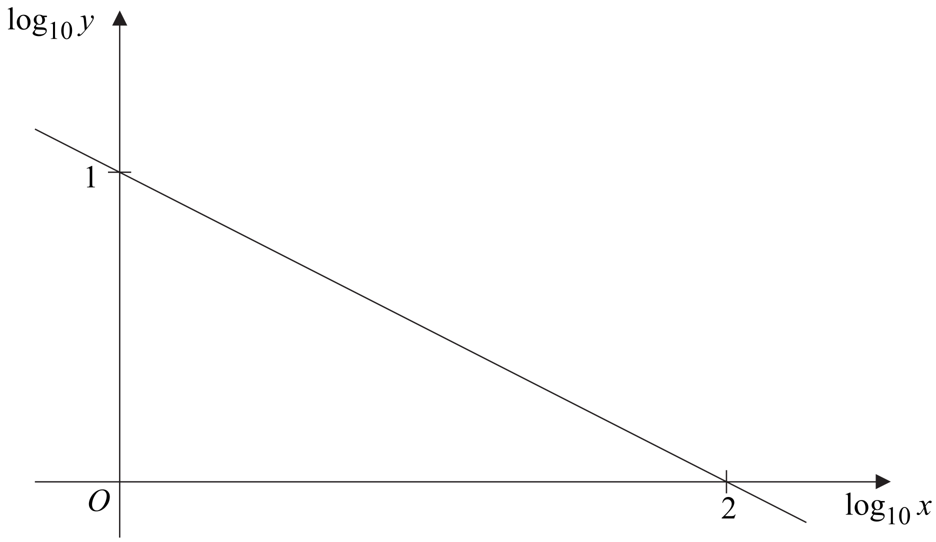
4 The variables x and y are related by an equation of the form

$$y = ax^b$$

where a and b are constants.

(a) Using logarithms to base 10, reduce the relation $y = ax^b$ to a linear law connecting $\log_{10} x$ and $\log_{10} y$. *(2 marks)*

(b) The diagram shows the linear graph that results from plotting $\log_{10} y$ against $\log_{10} x$.



Find the values of a and b. *(4 marks)*

5 A curve has equation

$$y = \frac{x}{x^2 - 1}$$

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) Sketch the curve.

(You are given that the curve has no stationary points.) *(4 marks)*

(c) Solve the inequality

$$\frac{x}{x^2 - 1} > 0$$ *(3 marks)*

6 (a) (i) Expand $(2r - 1)^2$. *(1 mark)*

(ii) Hence show that

$$\sum_{r=1}^{n}(2r - 1)^2 = \tfrac{1}{3}n(4n^2 - 1)$$ *(5 marks)*

(b) Hence find the sum of the squares of the odd numbers between 100 and 200. *(4 marks)*

7 The function f is defined for all real numbers by

$$\mathrm{f}(x) = \sin\left(x + \frac{\pi}{6}\right)$$

(a) Find the general solution of the equation $\mathrm{f}(x) = 0$. *(3 marks)*

(b) The quadratic function g is defined for all real numbers by

$$\mathrm{g}(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}x - \frac{1}{4}x^2$$

It can be shown that $\mathrm{g}(x)$ gives a good approximation to $\mathrm{f}(x)$ for small values of x.

(i) Show that $\mathrm{g}(0.05)$ and $\mathrm{f}(0.05)$ are identical when rounded to four decimal places. *(2 marks)*

(ii) A chord joins the points on the curve $y = \mathrm{g}(x)$ for which $x = 0$ and $x = h$. Find an expression in terms of h for the gradient of this chord. *(2 marks)*

(iii) Using your answer to part (b)(ii), find the value of $\mathrm{g}'(0)$. *(1 mark)*

8 A curve C has equation

$$\frac{x^2}{25} - \frac{y^2}{9} = 1$$

(a) Find the y-coordinates of the points on C for which $x = 10$, giving each answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(b) Sketch the curve C, indicating the coordinates of any points where the curve intersects the coordinate axes. *(3 marks)*

(c) Write down the equation of the tangent to C at the point where C intersects the positive x-axis. *(1 mark)*

(d) (i) Show that, if the line $y = x - 4$ intersects C, the x-coordinates of the points of intersection must satisfy the equation

$$16x^2 - 200x + 625 = 0$$ *(3 marks)*

(ii) Solve this equation and hence state the relationship between the line $y = x - 4$ and the curve C. *(2 marks)*

END OF QUESTIONS

General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Friday 25 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 7 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 It is given that $z_1 = 2 + \mathrm{i}$ and that $z_1{}^*$ is the complex conjugate of z_1.

Find the real numbers x and y such that

$$x + 3\mathrm{i}y = z_1 + 4\mathrm{i}z_1{}^*$$

(4 marks)

2 A curve satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2^x$$

Starting at the point $(1, 4)$ on the curve, use a step-by-step method with a step length of 0.01 to estimate the value of y at $x = 1.02$. Give your answer to six significant figures. *(5 marks)*

3 Find the general solution of the equation

$$\tan 4\left(x - \frac{\pi}{8}\right) = 1$$

giving your answer in terms of π. *(5 marks)*

4 (a) Find

$$\sum_{r=1}^{n} (r^3 - 6r)$$

expressing your answer in the form

$$kn(n+1)(n+p)(n+q)$$

where k is a fraction and p and q are integers. *(5 marks)*

(b) It is given that

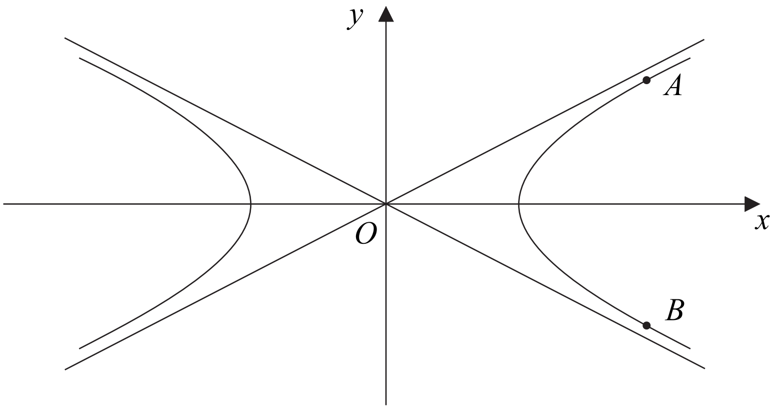
$$S = \sum_{r=1}^{1000} (r^3 - 6r)$$

Without calculating the value of S, show that S is a multiple of 2008. *(2 marks)*

5 The diagram shows the hyperbola

$$\frac{x^2}{4} - y^2 = 1$$

and its asymptotes.



(a) Write down the equations of the two asymptotes. *(2 marks)*

(b) The points on the hyperbola for which $x = 4$ are denoted by A and B.

Find, in surd form, the y-coordinates of A and B. *(2 marks)*

(c) The hyperbola and its asymptotes are translated by two units in the positive y direction.

Write down:

(i) the y-coordinates of the image points of A and B under this translation; *(1 mark)*

(ii) the equations of the hyperbola and the asymptotes after the translation. *(3 marks)*

Turn over for the next question

6 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \sqrt{3} & 3 \\ 3 & -\sqrt{3} \end{bmatrix}$$

(a) (i) Show that

$$\mathbf{M}^2 = p\mathbf{I}$$

where p is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(3 marks)*

(ii) Show that the matrix $\mathbf{M}$ can be written in the form

$$q\begin{bmatrix} \cos 60° & \sin 60° \\ \sin 60° & -\cos 60° \end{bmatrix}$$

where q is a real number. Give the value of q in surd form. *(3 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement and a reflection.

Find:

(i) the scale factor of the enlargement; *(1 mark)*

(ii) the equation of the mirror line of the reflection. *(1 mark)*

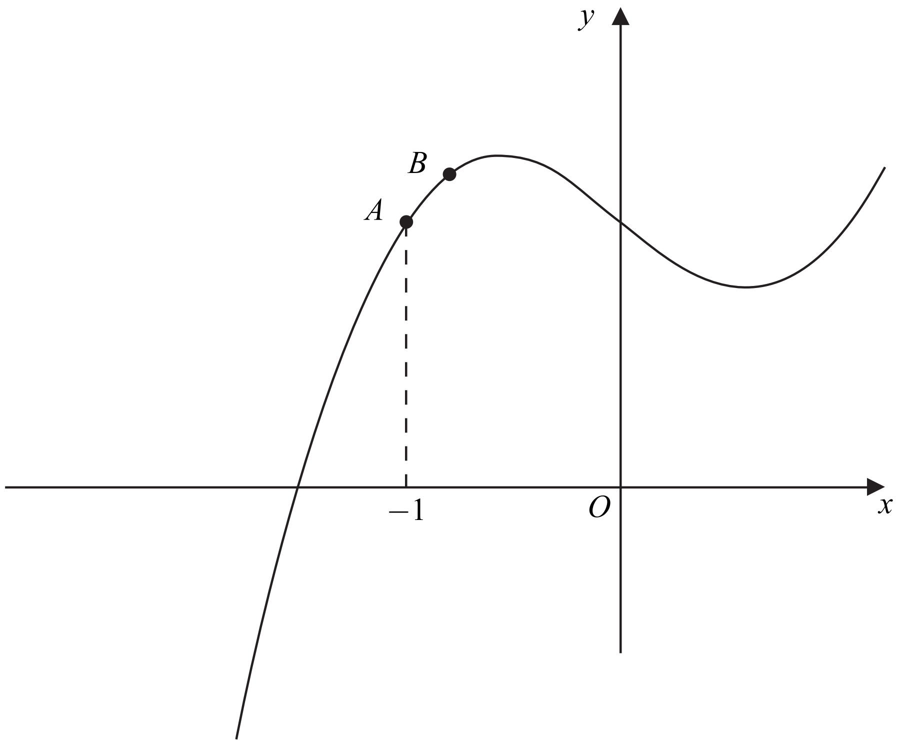
(c) Describe fully the geometrical transformation represented by $\mathbf{M}^4$. *(2 marks)*

7 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows the curve

$$y = x^3 - x + 1$$

The points A and B on the curve have x-coordinates -1 and $-1 + h$ respectively.



(a) (i) Show that the y-coordinate of the point B is

$$1 + 2h - 3h^2 + h^3$$ *(3 marks)*

(ii) Find the gradient of the chord AB in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(3 marks)*

(iii) Explain how your answer to part (a)(ii) can be used to find the gradient of the tangent to the curve at A. State the value of this gradient. *(2 marks)*

(b) The equation $x^3 - x + 1 = 0$ has one real root, α.

(i) Taking $x_1 = -1$ as a first approximation to α, use the Newton-Raphson method to find a second approximation, x_2, to α. *(2 marks)*

(ii) On **Figure 1**, draw a straight line to illustrate the Newton-Raphson method as used in part (b)(i). Show the points $(x_2, 0)$ and $(\alpha, 0)$ on your diagram. *(2 marks)*

Turn over ▶

8 (a) (i) It is given that α and β are the roots of the equation

$$x^2 - 2x + 4 = 0$$

Without solving this equation, show that α^3 and β^3 are the roots of the equation

$$x^2 + 16x + 64 = 0$$ *(6 marks)*

(ii) State, giving a reason, whether the roots of the equation

$$x^2 + 16x + 64 = 0$$

are real and equal, real and distinct, or non-real. *(2 marks)*

(b) Solve the equation

$$x^2 - 2x + 4 = 0$$ *(2 marks)*

(c) Use your answers to parts (a) and (b) to show that

$$(1 + \mathrm{i}\sqrt{3})^3 = (1 - \mathrm{i}\sqrt{3})^3$$ *(2 marks)*

9 A curve C has equation

$$y = \frac{2}{x(x-4)}$$

(a) Write down the equations of the three asymptotes of C. *(3 marks)*

(b) The curve C has one stationary point. By considering an appropriate quadratic equation, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(6 marks)*

(c) Sketch the curve C. *(3 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Insert

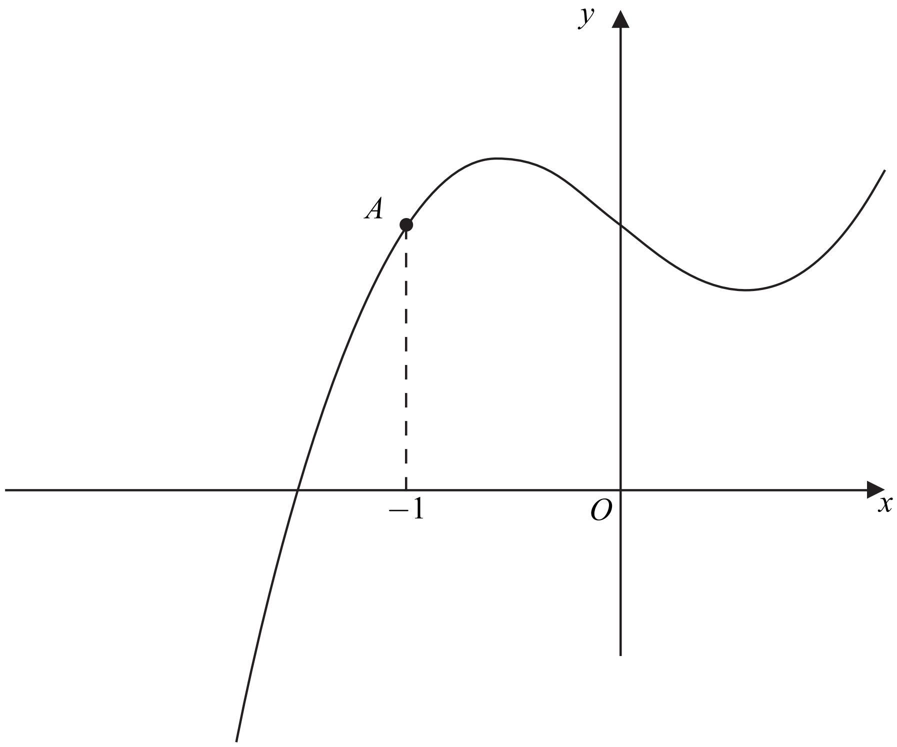
Insert for use in **Question 7**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Figure 1 (for use in Question 7)



General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Thursday 15 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A curve passes through the point $(0, 1)$ and satisfies the differential equation

$$\frac{dy}{dx} = \sqrt{1 + x^2}$$

Starting at the point $(0, 1)$, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 0.4$. Give your answer to five decimal places. *(5 marks)*

2 The complex number $2 + 3i$ is a root of the quadratic equation

$$x^2 + bx + c = 0$$

where b and c are real numbers.

(a) Write down the other root of this equation. *(1 mark)*

(b) Find the values of b and c. *(4 marks)*

3 Find the general solution of the equation

$$\tan\left(\frac{\pi}{2} - 3x\right) = \sqrt{3}$$ *(5 marks)*

4 It is given that

$$S_n = \sum_{r=1}^{n} (3r^2 - 3r + 1)$$

(a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = n^3$. *(5 marks)*

(b) Hence show that $\sum_{r=n+1}^{2n} (3r^2 - 3r + 1) = kn^3$ for some integer k. *(2 marks)*

5 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} k & k \\ k & -k \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -k & k \\ k & k \end{bmatrix}$$

where k is a constant.

(a) Find, in terms of k:

(i) $\mathbf{A} + \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{A}^2$. *(2 marks)*

(b) Show that $(\mathbf{A} + \mathbf{B})^2 = \mathbf{A}^2 + \mathbf{B}^2$. *(4 marks)*

(c) It is now given that $k = 1$.

(i) Describe the geometrical transformation represented by the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) The matrix **A** represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(3 marks)*

6 A curve has equation

$$y = \frac{(x-1)(x-3)}{x(x-2)}$$

(a) (i) Write down the equations of the three asymptotes of this curve. *(3 marks)*

(ii) State the coordinates of the points at which the curve intersects the x-axis. *(1 mark)*

(iii) Sketch the curve.

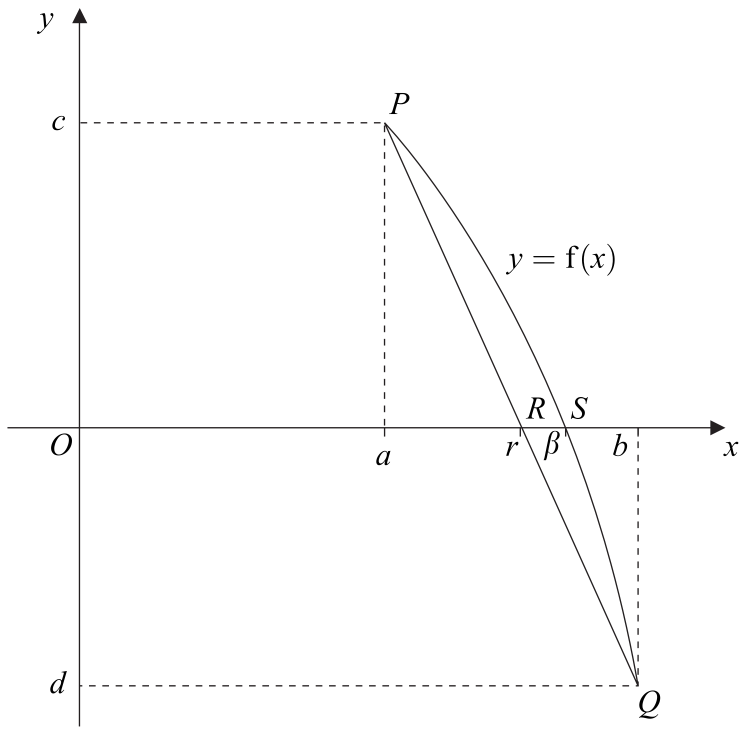
(You are given that the curve has no stationary points.) *(4 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{(x-1)(x-3)}{x(x-2)} < 0$$ *(2 marks)*

Turn over ▶

7 The points $P(a, c)$ and $Q(b, d)$ lie on the curve with equation $y = \mathrm{f}(x)$. The straight line PQ intersects the x-axis at the point $R(r, 0)$. The curve $y = \mathrm{f}(x)$ intersects the x-axis at the point $S(\beta, 0)$.



(a) Show that

$$r = a + c\left(\frac{b-a}{c-d}\right)$$ *(4 marks)*

(b) Given that

$$a = 2,\ b = 3 \text{ and } \mathrm{f}(x) = 20x - x^4$$

(i) find the value of r; *(3 marks)*

(ii) show that $\beta - r \approx 0.18$. *(3 marks)*

8 For each of the following improper integrals, find the value of the integral **or** explain why it does not have a value:

(a) $\displaystyle\int_1^{\infty} x^{-\frac{3}{4}}\,\mathrm{d}x$; *(3 marks)*

(b) $\displaystyle\int_1^{\infty} x^{-\frac{5}{4}}\,\mathrm{d}x$; *(3 marks)*

(c) $\displaystyle\int_1^{\infty} \left(x^{-\frac{3}{4}} - x^{-\frac{5}{4}}\right)\mathrm{d}x$. *(1 mark)*

9 A hyperbola H has equation

$$x^2 - \frac{y^2}{2} = 1$$

(a) Find the equations of the two asymptotes of H, giving each answer in the form $y = mx$. *(2 marks)*

(b) Draw a sketch of the two asymptotes of H, using roughly equal scales on the two coordinate axes. Using the same axes, sketch the hyperbola H. *(3 marks)*

(c) (i) Show that, if the line $y = x + c$ intersects H, the x-coordinates of the points of intersection must satisfy the equation

$$x^2 - 2cx - (c^2 + 2) = 0$$ *(4 marks)*

(ii) Hence show that the line $y = x + c$ intersects H in two distinct points, whatever the value of c. *(2 marks)*

(iii) Find, in terms of c, the y-coordinates of these two points. *(3 marks)*

END OF QUESTIONS

General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MFP1

Unit Further Pure 1

Wednesday 13 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 6 (enclosed).

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
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- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The quadratic equation

$$3x^2 - 6x + 1 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^3 + \beta^3 = 6$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. *(4 marks)*

2 The complex number z is defined by

$$z = 1 + \mathrm{i}$$

(a) Find the value of z^2, giving your answer in its simplest form. *(2 marks)*

(b) Hence show that $z^8 = 16$. *(2 marks)*

(c) Show that $(z^*)^2 = -z^2$. *(2 marks)*

3 Find the general solution of the equation

$$\sin\left(4x + \frac{\pi}{4}\right) = 1$$ *(4 marks)*

4 It is given that

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 1 \end{bmatrix}$$

and that $\mathbf{I}$ is the 2×2 identity matrix.

(a) Show that $(\mathbf{A} - \mathbf{I})^2 = k\mathbf{I}$ for some integer k. *(3 marks)*

(b) Given further that

$$\mathbf{B} = \begin{bmatrix} 1 & 3 \\ p & 1 \end{bmatrix}$$

find the integer p such that

$$(\mathbf{A} - \mathbf{B})^2 = (\mathbf{A} - \mathbf{I})^2$$ *(4 marks)*

5 (a) Explain why $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,\mathrm{d}x$ is an improper integral. *(1 mark)*

(b) For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

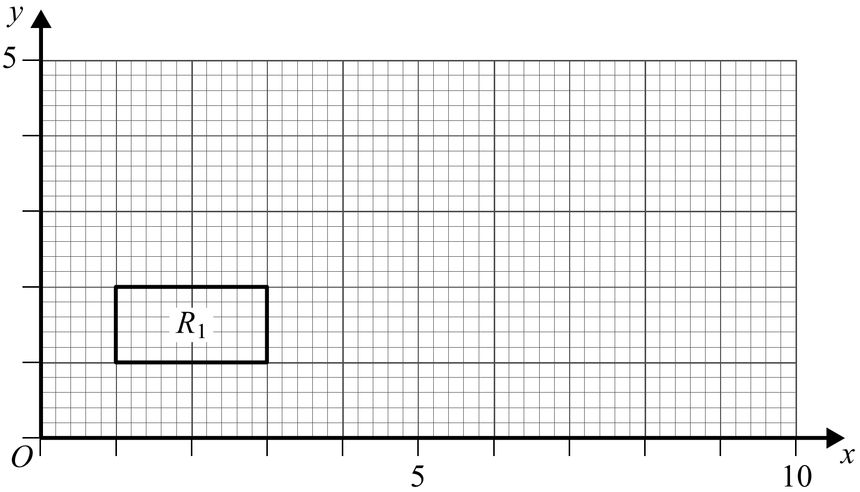
(i) $\int_0^{\frac{1}{16}} x^{-\frac{1}{2}}\,\mathrm{d}x$; *(3 marks)*

(ii) $\int_0^{\frac{1}{16}} x^{-\frac{5}{4}}\,\mathrm{d}x$. *(3 marks)*

Turn over for the next question

6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The diagram shows a rectangle R_1 .



(a) The rectangle R_1 is mapped onto a second rectangle, R_2 , by a transformation with matrix $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$.

(i) Calculate the coordinates of the vertices of the rectangle R_2 . *(2 marks)*

(ii) On **Figure 1**, draw the rectangle R_2 . *(1 mark)*

(b) The rectangle R_2 is rotated through 90° clockwise about the origin to give a third rectangle, R_3 .

(i) On **Figure 1**, draw the rectangle R_3 . *(2 marks)*

(ii) Write down the matrix of the rotation which maps R_2 onto R_3 . *(1 mark)*

(c) Find the matrix of the transformation which maps R_1 onto R_3 . *(2 marks)*

7 A curve C has equation $y = \dfrac{1}{(x-2)^2}$.

(a) (i) Write down the equations of the asymptotes of the curve C. *(2 marks)*

(ii) Sketch the curve C. *(2 marks)*

(b) The line $y = x - 3$ intersects the curve C at a point which has x-coordinate α.

(i) Show that α lies within the interval $3 < x < 4$. *(2 marks)*

(ii) Starting from the interval $3 < x < 4$, use interval bisection twice to obtain an interval of width 0.25 within which α must lie. *(3 marks)*

8 (a) Show that

$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r$$

can be expressed in the form

$$kn(n+1)(an^2 + bn + c)$$

where k is a rational number and a, b and c are integers. *(4 marks)*

(b) Show that there is exactly one positive integer n for which

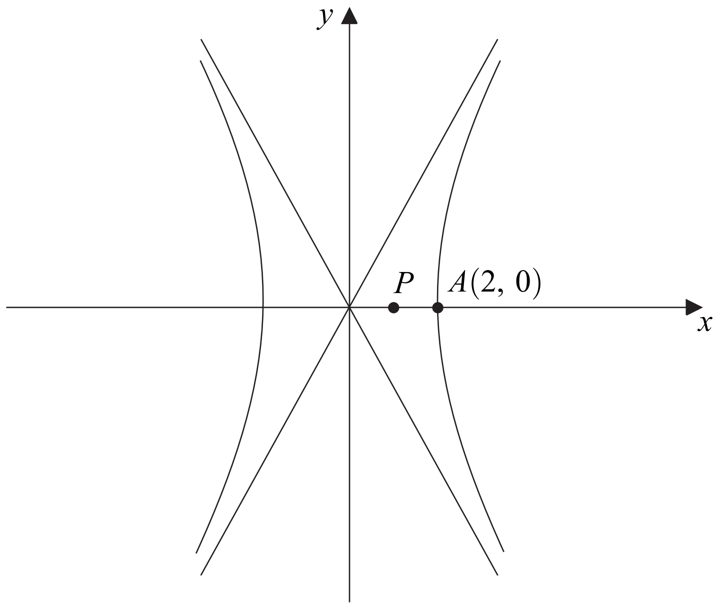
$$\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r = 8\sum_{r=1}^{n} r^2$$ *(5 marks)*

Turn over for the next question

9 The diagram shows the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and its asymptotes.



The constants a and b are positive integers.

The point A on the hyperbola has coordinates $(2, 0)$.

The equations of the asymptotes are $y = 2x$ and $y = -2x$.

(a) Show that $a = 2$ and $b = 4$. *(4 marks)*

(b) The point P has coordinates $(1, 0)$. A straight line passes through P and has gradient m. Show that, if this line intersects the hyperbola, the x-coordinates of the points of intersection satisfy the equation

$$(m^2 - 4)x^2 - 2m^2x + (m^2 + 16) = 0$$ *(4 marks)*

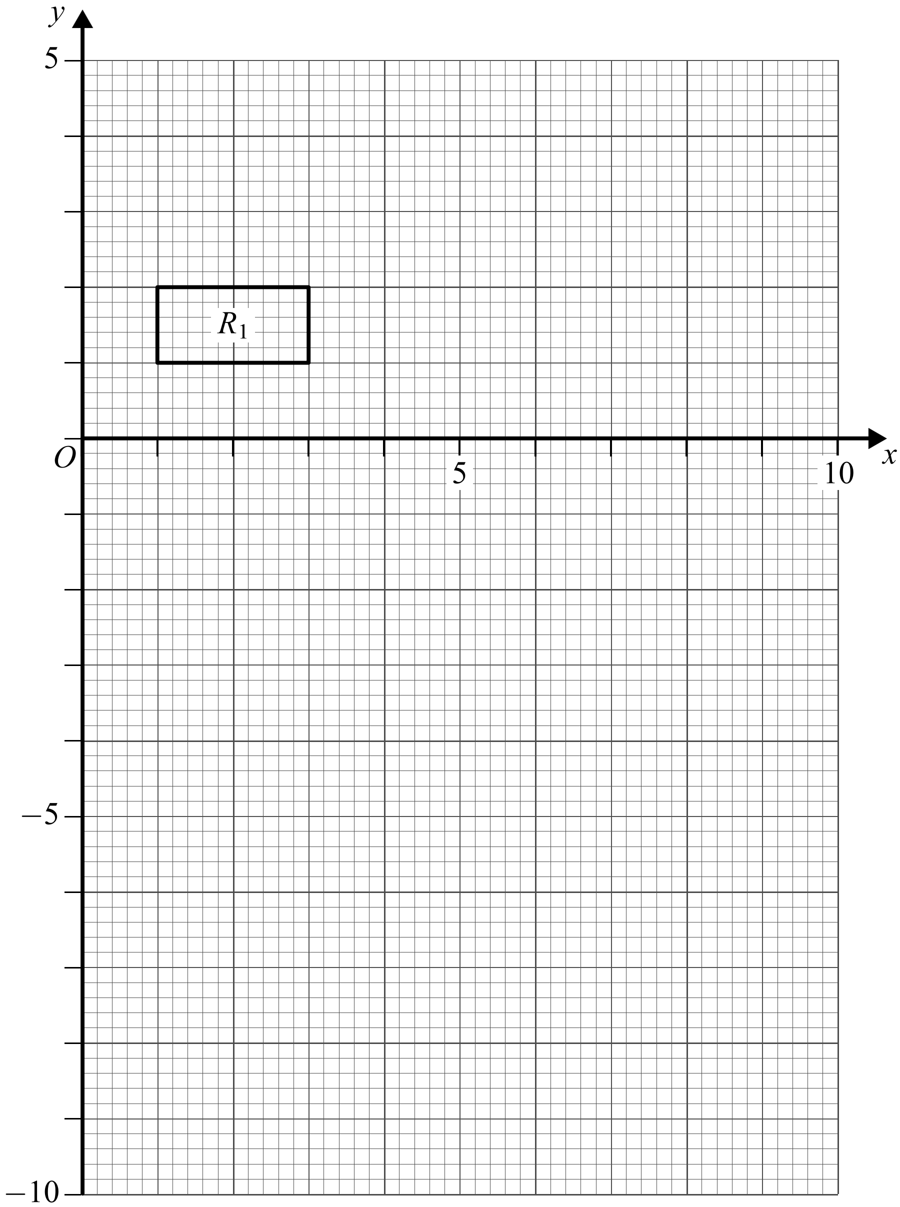
(c) Show that this equation has equal roots if $3m^2 = 16$. *(3 marks)*

(d) There are two tangents to the hyperbola which pass through P. Find the coordinates of the points at which these tangents touch the hyperbola.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

END OF QUESTIONS

Figure 1 (for use in Question 6)



General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MFP1

Unit Further Pure 1

Friday 14 January 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The quadratic equation $x^2 - 6x + 18 = 0$ has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find a quadratic equation, with integer coefficients, which has roots α^2 and β^2. *(4 marks)*

(c) Hence find the values of α^2 and β^2. *(1 mark)*

---

2 (a) Find, in terms of p and q, the value of the integral $\displaystyle\int_p^q \frac{2}{x^3}\,\mathrm{d}x$. *(3 marks)*

(b) Show that only one of the following improper integrals has a finite value, and find that value:

(i) $\displaystyle\int_0^2 \frac{2}{x^3}\,\mathrm{d}x$;

(ii) $\displaystyle\int_2^\infty \frac{2}{x^3}\,\mathrm{d}x$. *(3 marks)*

---

3 (a) Write down the 2×2 matrix corresponding to each of the following transformations:

(i) a rotation about the origin through 90° clockwise; *(1 mark)*

(ii) a rotation about the origin through 180°. *(1 mark)*

(b) The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 1 \\ -4 & 3 \end{bmatrix}$$

(i) Calculate the matrix $\mathbf{AB}$. *(2 marks)*

(ii) Show that $(\mathbf{A} + \mathbf{B})^2 = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, for some integer k. *(3 marks)*

(c) Describe the single geometrical transformation, or combination of two geometrical transformations, represented by each of the following matrices:

(i) $\mathbf{A} + \mathbf{B}$; *(2 marks)*

(ii) $(\mathbf{A} + \mathbf{B})^2$; *(2 marks)*

(iii) $(\mathbf{A} + \mathbf{B})^4$. *(2 marks)*

---

4 Find the general solution of the equation

$$\sin\left(4x - \frac{2\pi}{3}\right) = -\frac{1}{2}$$

giving your answer in terms of π. *(6 marks)*

---

5 (a) It is given that $z_1 = \frac{1}{2} - \mathrm{i}$.

(i) Calculate the value of $z_1{}^2$, giving your answer in the form $a + b\mathrm{i}$. *(2 marks)*

(ii) Hence verify that z_1 is a root of the equation

$$z^2 + z^* + \frac{1}{4} = 0$$ *(2 marks)*

(b) Show that $z_2 = \frac{1}{2} + \mathrm{i}$ also satisfies the equation in part **(a)(ii)**. *(2 marks)*

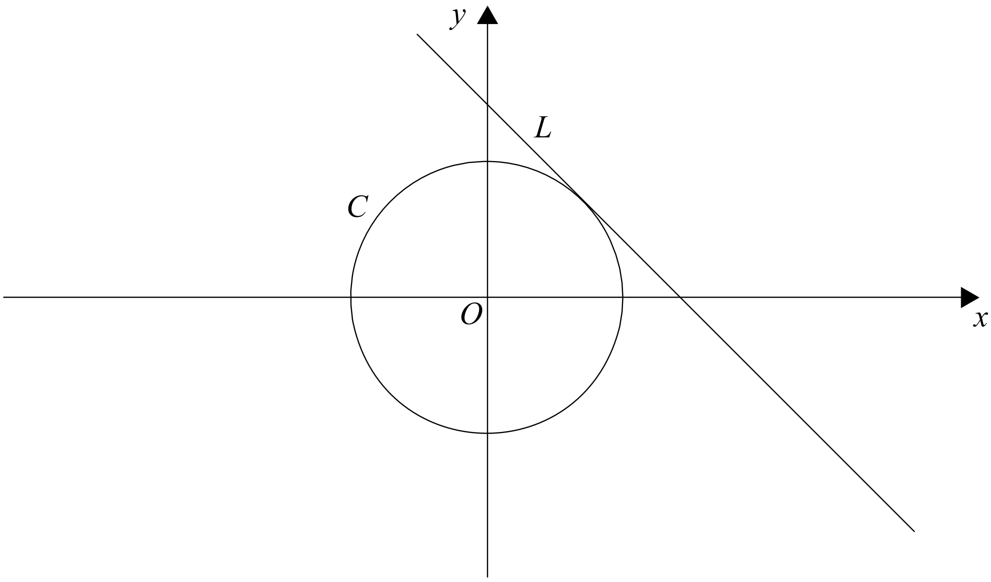
(c) Show that the equation in part **(a)(ii)** has two equal **real** roots. *(2 marks)*

---

6 The diagram shows a circle C and a line L, which is the tangent to C at the point $(1, 1)$. The equations of C and L are

$$x^2 + y^2 = 2 \quad \text{and} \quad x + y = 2$$

respectively.



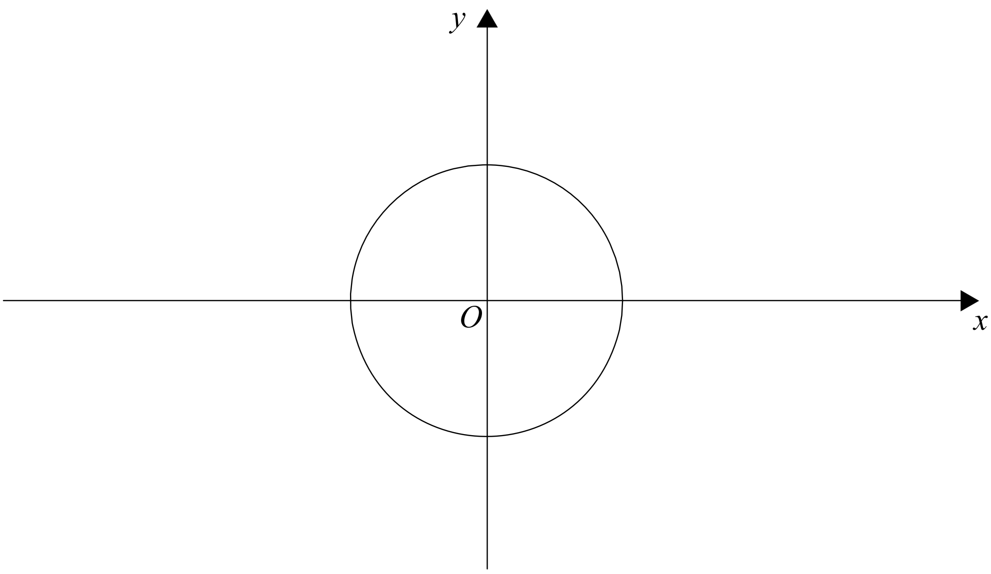
The circle C is now transformed by a stretch with scale factor 2 parallel to the x-axis. The image of C under this stretch is an ellipse E.

(a) **On the diagram below**, sketch the ellipse E, indicating the coordinates of the points where it intersects the coordinate axes. *(4 marks)*

(b) Find equations of:

(i) the ellipse E; *(2 marks)*

(ii) the tangent to E at the point $(2, 1)$. *(2 marks)*



7 A graph has equation

$$y = \frac{x-4}{x^2+9}$$

(a) Explain why the graph has no vertical asymptote and give the equation of the horizontal asymptote. *(2 marks)*

(b) Show that, if the line $y = k$ intersects the graph, the x-coordinates of the points of intersection of the line with the graph must satisfy the equation

$$kx^2 - x + (9k+4) = 0$$ *(2 marks)*

(c) Show that this equation has real roots if $-\frac{1}{2} \leqslant k \leqslant \frac{1}{18}$. *(5 marks)*

(d) Hence find the coordinates of the two stationary points on the curve.

(No credit will be given for methods involving differentiation.) *(6 marks)*

---

8 (a) The equation

$$x^3 + 2x^2 + x - 100\,000 = 0$$

has one real root. Taking $x_1 = 50$ as a first approximation to this root, use the Newton-Raphson method to find a second approximation, x_2, to the root. *(3 marks)*

(b) (i) Given that $S_n = \sum_{r=1}^{n} r(3r+1)$, use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$S_n = n(n+1)^2$$ *(5 marks)*

(ii) The lowest integer n for which $S_n > 100\,000$ is denoted by N.

Show that

$$N^3 + 2N^2 + N - 100\,000 > 0$$ *(1 mark)*

(c) Find the value of N, justifying your answer. *(3 marks)*

General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MFP1

Unit Further Pure 1

Tuesday 17 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = \frac{17}{4}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$ *(5 marks)*

---

2 Show that only one of the following improper integrals has a finite value, and find that value:

(a) $\displaystyle\int_8^\infty x^{-\frac{2}{3}}\,dx$;

(b) $\displaystyle\int_8^\infty x^{-\frac{4}{3}}\,dx$. *(5 marks)*

---

3 (a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 9 = 0$; *(1 mark)*

(ii) $(x+2)^2 + 9 = 0$. *(1 mark)*

(b) (i) Expand $(1+x)^3$. *(1 mark)*

(ii) Express $(1+2i)^3$ in the form $a + bi$. *(3 marks)*

(iii) Given that $z = 1 + 2i$, find the value of

$$z^* - z^3$$ *(2 marks)*

4 (a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r^2(4r-3) = kn(n+1)(2n^2-1)$$

where k is a constant. *(5 marks)*

(b) Hence evaluate

$$\sum_{r=20}^{40} r^2(4r-3)$$ *(2 marks)*

---

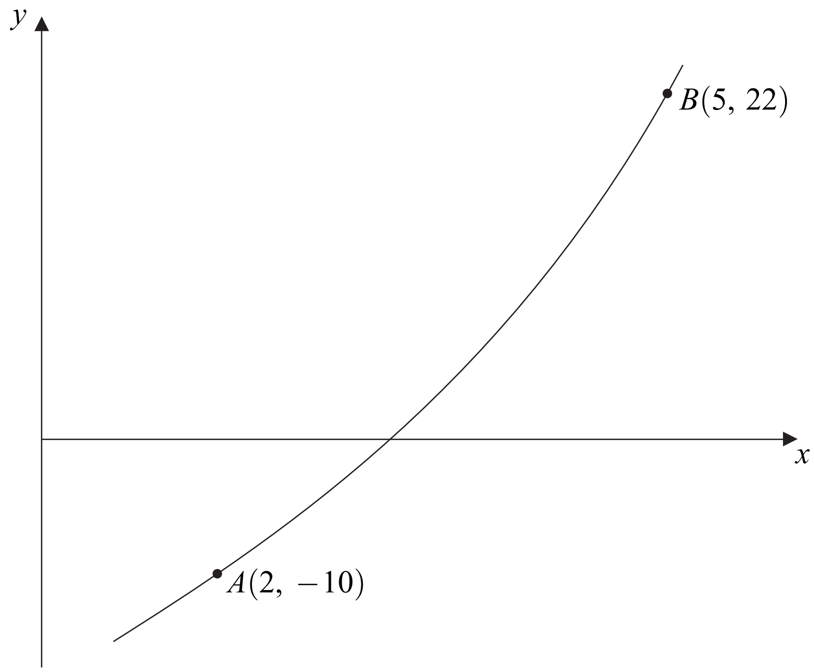
5 The diagram below (not to scale) shows a part of a curve $y = \mathrm{f}(x)$ which passes through the points $A(2, -10)$ and $B(5, 22)$.

(a) (i) On the diagram, draw a line which illustrates the method of linear interpolation for solving the equation $\mathrm{f}(x) = 0$. The point of intersection of this line with the x-axis should be labelled P. *(1 mark)*

(ii) Calculate the x-coordinate of P. Give your answer to one decimal place. *(3 marks)*

(b) (i) On the same diagram, draw a line which illustrates the Newton–Raphson method for solving the equation $\mathrm{f}(x) = 0$, with initial value $x_1 = 2$. The point of intersection of this line with the x-axis should be labelled Q. *(1 mark)*

(ii) Given that the gradient of the curve at A is 8, calculate the x-coordinate of Q. Give your answer as an exact decimal. *(2 marks)*



6 Find the general solution of each of the following equations:

(a) $\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{\sqrt{3}}$; *(4 marks)*

(b) $\tan^2\left(\frac{x}{2} - \frac{\pi}{4}\right) = \frac{1}{3}$. *(3 marks)*

---

7 A hyperbola H has equation

$$\frac{x^2}{9} - y^2 = 1$$

(a) Find the equations of the asymptotes of H. *(1 mark)*

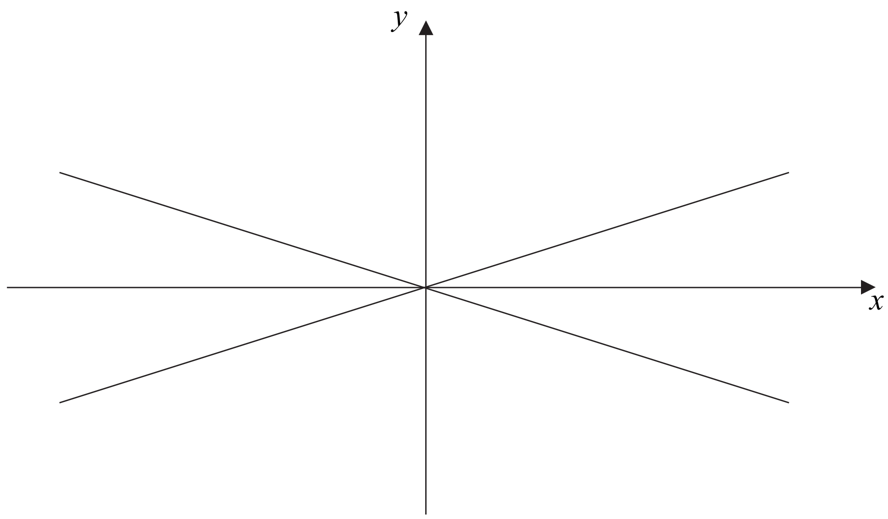
(b) The asymptotes of H are shown in the diagram opposite. On the same diagram, sketch the hyperbola H. Indicate on your sketch the coordinates of the points of intersection of H with the coordinate axes. *(3 marks)*

(c) The hyperbola H is now translated by the vector $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$.

(i) Write down the equation of the translated curve. *(2 marks)*

(ii) Calculate the coordinates of the two points of intersection of the translated curve with the line $y = x$. *(4 marks)*

(d) From your answers to part **(c)(ii)**, **deduce** the coordinates of the points of intersection of the original hyperbola H with the line $y = x - 3$. *(2 marks)*



8 The diagram below shows a rectangle R_1 which has vertices (0, 0), (3, 0), (3, 2) and (0, 2).

(a) On the diagram, draw:

(i) the image R_2 of R_1 under a rotation through 90° clockwise about the origin; *(1 mark)*

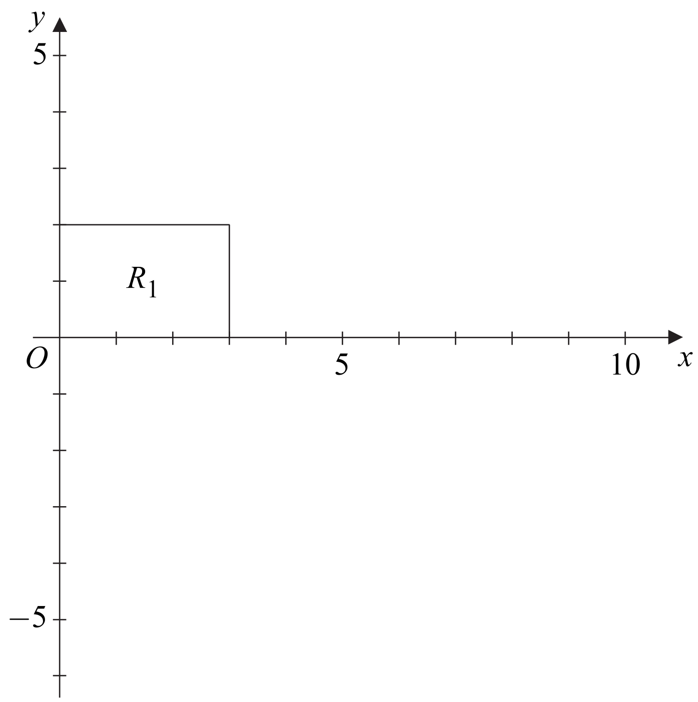
(ii) the image R_3 of R_2 under the transformation which has matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$ *(3 marks)*

(b) Find the matrix of:

(i) the rotation which maps R_1 onto R_2; *(1 mark)*

(ii) the combined transformation which maps R_1 onto R_3. *(3 marks)*



Turn over ▶

9 A curve has equation

$$y = \frac{x}{x-1}$$

(a) Find the equations of the asymptotes of this curve. *(2 marks)*

(b) Given that the line $y = -4x + c$ intersects the curve, show that the x-coordinates of the points of intersection must satisfy the equation

$$4x^2 - (c+3)x + c = 0$$ *(3 marks)*

(c) It is given that the line $y = -4x + c$ is a tangent to the curve.

(i) Find the two possible values of c.

(No credit will be given for methods involving differentiation.) *(3 marks)*

(ii) For each of the two values found in part **(c)(i)**, find the coordinates of the point where the line touches the curve. *(4 marks)*

General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MFP1

Unit Further Pure 1

Friday 18 January 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A curve passes through the point $(1, 3)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{1+x^3}$$

Starting at the point $(1, 3)$, use a step-by-step method with a step length of 0.1 to estimate the value of y at $x = 1.2$. Give your answer to four decimal places. *(5 marks)*

---

2 (a) Solve the equation $w^2 + 6w + 34 = 0$, giving your answers in the form $p + q\mathrm{i}$, where p and q are integers. *(3 marks)*

(b) It is given that $z = \mathrm{i}(1+\mathrm{i})(2+\mathrm{i})$.

(i) Express z in the form $a + b\mathrm{i}$, where a and b are integers. *(3 marks)*

(ii) Find integers m and n such that $z + mz^* = n\mathrm{i}$. *(3 marks)*

---

3 (a) Find the general solution of the equation

$$\sin\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(6 marks)*

(b) Use your general solution to find the exact value of the greatest solution of this equation which is less than 6π. *(2 marks)*

---

4 Show that the improper integral $\int_{25}^{\infty} \frac{1}{x\sqrt{x}}\,\mathrm{d}x$ has a finite value and find that value. *(4 marks)*

5 The roots of the quadratic equation

$$x^2 + 2x - 5 = 0$$

are α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. *(2 marks)*

(b) Calculate the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots $\alpha^3\beta + 1$ and $\alpha\beta^3 + 1$. *(5 marks)*

---

6 (a) The matrix $\mathbf{X}$ is defined by $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$.

(i) Given that $\mathbf{X}^2 = \begin{bmatrix} m & 2 \\ 3 & 6 \end{bmatrix}$, find the value of m. *(1 mark)*

(ii) Show that $\mathbf{X}^3 - 7\mathbf{X} = n\mathbf{I}$, where n is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(4 marks)*

(b) It is given that $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

(i) Describe the geometrical transformation represented by $\mathbf{A}$. *(1 mark)*

(ii) The matrix $\mathbf{B}$ represents an anticlockwise rotation through $45°$ about the origin.

Show that $\mathbf{B} = k\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, where k is a surd. *(2 marks)*

(iii) Find the image of the point $P(-1, 2)$ under an anticlockwise rotation through $45°$ about the origin, followed by the transformation represented by $\mathbf{A}$. *(4 marks)*

7 The variables y and x are related by an equation of the form

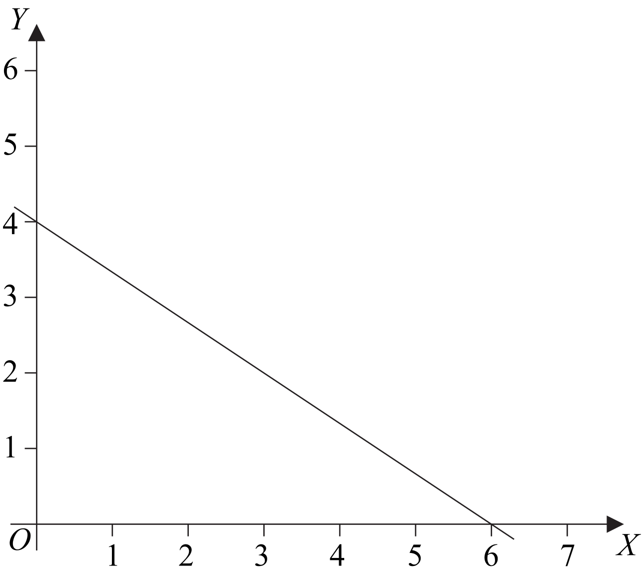
$$y = ax^n$$

where a and n are constants.

Let $Y = \log_{10} y$ and $X = \log_{10} x$.

(a) Show that there is a linear relationship between Y and X. *(3 marks)*

(b) The graph of Y against X is shown in the diagram.



Find the value of n and the value of a. *(4 marks)*

---

8 (a) Show that

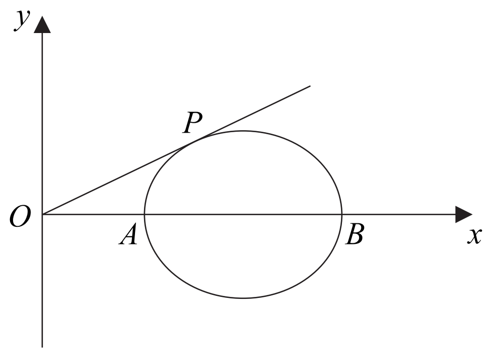
$$\sum_{r=1}^{n} 2r(2r^2 - 3r - 1) = n(n+p)(n+q)^2$$

where p and q are integers to be found. *(6 marks)*

(b) Hence find the value of

$$\sum_{r=11}^{20} 2r(2r^2 - 3r - 1)$$ *(2 marks)*

9 An ellipse is shown below.



The ellipse intersects the x-axis at the points A and B. The equation of the ellipse is

$$\frac{(x-4)^2}{4} + y^2 = 1$$

(a) Find the x-coordinates of A and B. *(2 marks)*

(b) The line $y = mx$ $(m > 0)$ is a tangent to the ellipse, with point of contact P.

(i) Show that the x-coordinate of P satisfies the equation

$$(1 + 4m^2)x^2 - 8x + 12 = 0$$ *(3 marks)*

(ii) Hence find the exact value of m. *(4 marks)*

(iii) Find the coordinates of P. *(4 marks)*

General Certificate of Education
June 2006
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Monday 12 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The quadratic equation

$$3x^2 - 6x + 2 = 0$$

has roots α and β.

(a) Write down the numerical values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) (i) Expand $(\alpha + \beta)^3$. *(1 mark)*

(ii) Show that $\alpha^3 + \beta^3 = 4$. *(3 marks)*

(c) Find a quadratic equation with roots α^3 and β^3, giving your answer in the form $px^2 + qx + r = 0$, where p, q and r are integers. *(3 marks)*

2 A curve satisfies the differential equation

$$\frac{dy}{dx} = \log_{10} x$$

Starting at the point (2, 3) on the curve, use a step-by-step method with a step length of 0.2 to estimate the value of y at $x = 2.4$. Give your answer to three decimal places. *(6 marks)*

3 Show that

$$\sum_{r=1}^{n} (r^2 - r) = kn(n+1)(n-1)$$

where k is a rational number. *(4 marks)*

4 Find, in **radians**, the general solution of the equation

$$\cos 3x = \frac{\sqrt{3}}{2}$$

giving your answers in terms of π. *(5 marks)*

5 The matrix $\mathbf{M}$ is defined by

$$\mathbf{M} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

(a) Find the matrix:

(i) $\mathbf{M}^2$; *(3 marks)*

(ii) $\mathbf{M}^4$. *(1 mark)*

(b) Describe fully the geometrical transformation represented by $\mathbf{M}$. *(2 marks)*

(c) Find the matrix $\mathbf{M}^{2006}$. *(3 marks)*

6 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Write down, in terms of x and y, an expression for

$$(z + \mathrm{i})^*$$

where $(z + \mathrm{i})^*$ denotes the complex conjugate of $(z + \mathrm{i})$. *(2 marks)*

(b) Solve the equation

$$(z + \mathrm{i})^* = 2\mathrm{i}z + 1$$

giving your answer in the form $a + b\mathrm{i}$. *(5 marks)*

Turn over for the next question

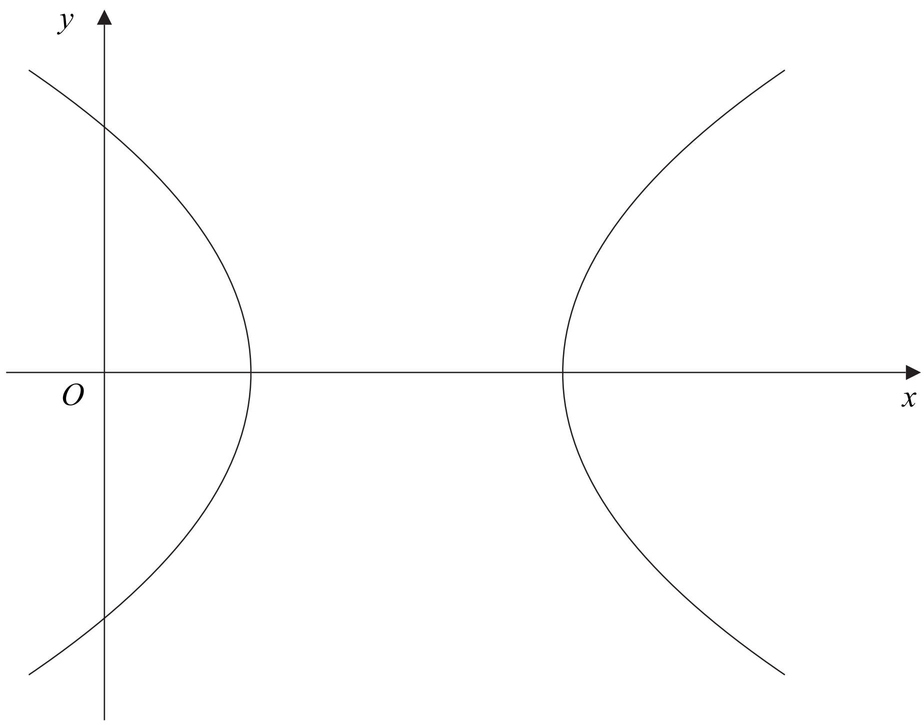
7 (a) Describe a geometrical transformation by which the hyperbola

$$x^2 - 4y^2 = 1$$

can be obtained from the hyperbola $x^2 - y^2 = 1$. *(2 marks)*

(b) The diagram shows the hyperbola H with equation

$$x^2 - y^2 - 4x + 3 = 0$$



By completing the square, describe a geometrical transformation by which the hyperbola H can be obtained from the hyperbola $x^2 - y^2 = 1$. *(4 marks)*

8 (a) The function f is defined for all real values of x by

$$f(x) = x^3 + x^2 - 1$$

(i) Express $f(1+h) - f(1)$ in the form

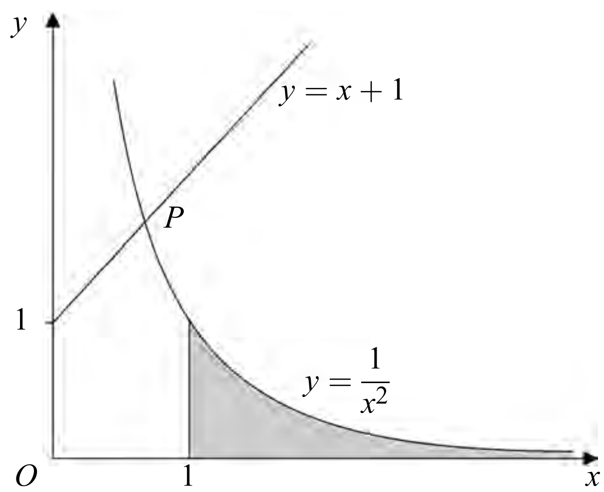
$$ph + qh^2 + rh^3$$

where p, q and r are integers. *(4 marks)*

(ii) Use your answer to part (a)(i) to find the value of $f'(1)$. *(2 marks)*

(b) The diagram shows the graphs of

$$y = \frac{1}{x^2} \quad \text{and} \quad y = x + 1 \quad \text{for} \quad x > 0$$



The graphs intersect at the point P.

(i) Show that the x-coordinate of P satisfies the equation $f(x) = 0$, where f is the function defined in part (a). *(1 mark)*

(ii) Taking $x_1 = 1$ as a first approximation to the root of the equation $f(x) = 0$, use the Newton–Raphson method to find a second approximation x_2 to the root. *(3 marks)*

(c) The region enclosed by the curve $y = \dfrac{1}{x^2}$, the line $x = 1$ and the x-axis is shaded on the diagram. By evaluating an improper integral, find the area of this region. *(3 marks)*

Turn over ▶

9 A curve C has equation

$$y = \frac{(x+1)(x-3)}{x(x-2)}$$

(a) (i) Write down the coordinates of the points where C intersects the x-axis. *(2 marks)*

(ii) Write down the equations of all the asymptotes of C. *(3 marks)*

(b) (i) Show that, if the line $y = k$ intersects C, then

$$(k-1)(k-4) \geqslant 0$$ *(5 marks)*

(ii) Given that there is only one stationary point on C, find the coordinates of this stationary point.

(No credit will be given for solutions based on differentiation.) *(3 marks)*

(c) Sketch the curve C. *(3 marks)*

END OF QUESTIONS

General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Wednesday 20 June 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables
- an insert for use in Questions 5 and 9 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 8 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

The matrix $\mathbf{M} = \mathbf{A} - 2\mathbf{B}$.

(a) Show that $\mathbf{M} = n\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, where n is a positive integer. *(2 marks)*

(b) The matrix $\mathbf{M}$ represents a combination of an enlargement of scale factor p and a reflection in a line L. State the value of p and write down the equation of L. *(2 marks)*

(c) Show that

$$\mathbf{M}^2 = q\mathbf{I}$$

where q is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

2 (a) Show that the equation

$$x^3 + x - 7 = 0$$

has a root between 1.6 and 1.8. *(3 marks)*

(b) Use interval bisection **twice**, starting with the interval in part (a), to give this root to one decimal place. *(4 marks)*

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$z - 3\mathrm{i}z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$z - 3\mathrm{i}z^* = 16$$ *(3 marks)*

4 The quadratic equation

$$2x^2 - x + 4 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{1}{4}$. *(2 marks)*

(c) Find a quadratic equation with integer coefficients such that the roots of the equation are

$$\frac{4}{\alpha} \text{ and } \frac{4}{\beta}$$ *(3 marks)*

5 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are known to be related by an equation of the form

$$y = ab^x$$

where a and b are constants.

The following approximate values of x and y have been found.

x	1	2	3	4
y	3.84	6.14	9.82	15.7

(a) Complete the table in **Figure 1**, showing values of x and Y, where $Y = \log_{10} y$.
Give each value of Y to three decimal places. *(2 marks)*

(b) Show that, if $y = ab^x$, then x and Y must satisfy an equation of the form

$$Y = mx + c$$ *(3 marks)*

(c) Draw on **Figure 2** a linear graph relating x and Y. *(2 marks)*

(d) Hence find estimates for the values of a and b. *(4 marks)*

Turn over ▶

6 Find the general solution of the equation

$$\sin\left(2x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(6 marks)*

7 A curve has equation

$$y = \frac{3x - 1}{x + 2}$$

(a) Write down the equations of the two asymptotes to the curve. *(2 marks)*

(b) Sketch the curve, indicating the coordinates of the points where the curve intersects the coordinate axes. *(5 marks)*

(c) Hence, or otherwise, solve the inequality

$$0 < \frac{3x - 1}{x + 2} < 3$$ *(2 marks)*

8 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\displaystyle\int_0^1 \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right) \mathrm{d}x$; *(4 marks)*

(b) $\displaystyle\int_0^1 \frac{x^{\frac{1}{3}} + x^{-\frac{1}{3}}}{x} \mathrm{d}x$. *(4 marks)*

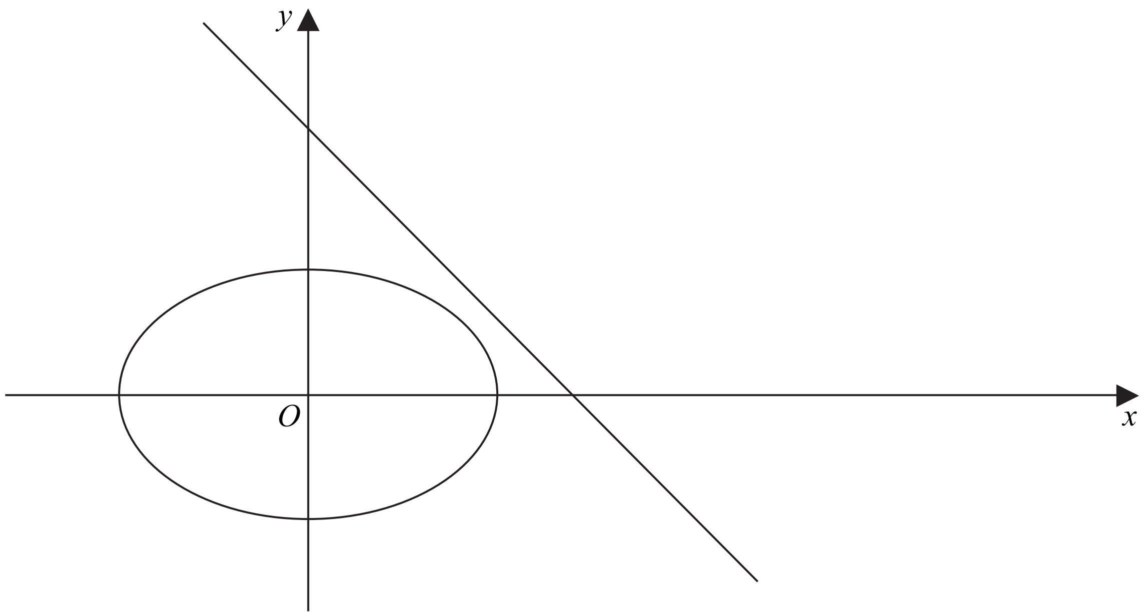
9 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows the curve with equation

$$\frac{x^2}{2} + y^2 = 1$$

and the straight line with equation

$$x + y = 2$$



(a) Write down the exact coordinates of the points where the curve with equation $\frac{x^2}{2} + y^2 = 1$ intersects the coordinate axes. *(2 marks)*

(b) The curve is translated k units in the positive x direction, where k is a constant. Write down, in terms of k, the equation of the curve after this translation. *(2 marks)*

(c) Show that, if the line $x + y = 2$ intersects the **translated** curve, the x-coordinates of the points of intersection must satisfy the equation

$$3x^2 - 2(k+4)x + (k^2 + 6) = 0$$ *(4 marks)*

(d) Hence find the two values of k for which the line $x + y = 2$ is a tangent to the translated curve. Give your answer in the form $p \pm \sqrt{q}$, where p and q are integers. *(4 marks)*

(e) On **Figure 3**, show the translated curves corresponding to these two values of k. *(3 marks)*

END OF QUESTIONS

Surname		Other Names										
Centre Number						Candidate Number						
Candidate Signature												

General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

ASSESSMENT and QUALIFICATIONS ALLIANCE

Insert

Insert for use in **Questions 5 and 9**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 5)

x	1	2	3	4
Y	0.584			

Figure 2 (for use in Question 5)

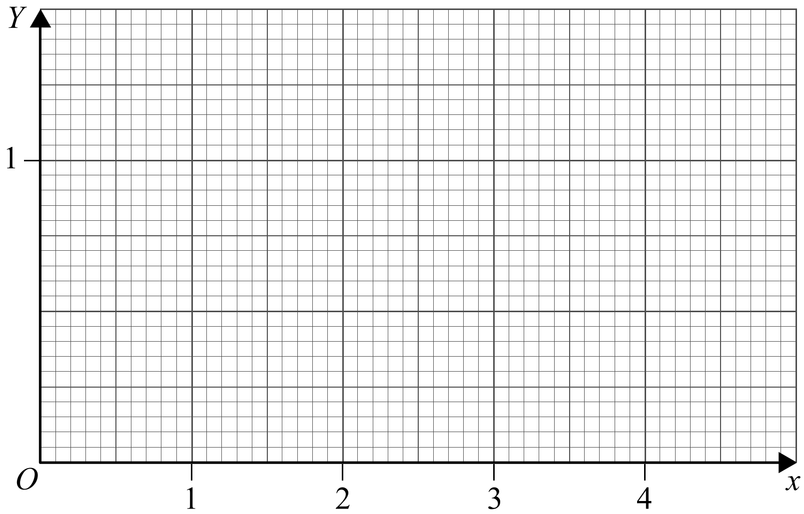
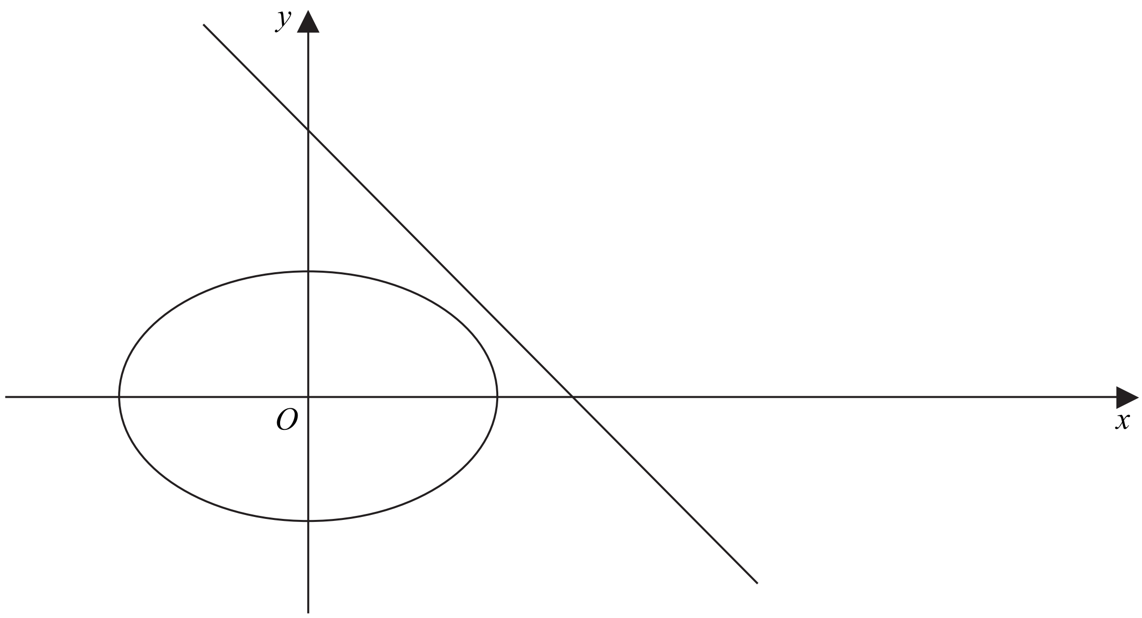


Figure 3 (for use in Question 9)



General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

Monday 16 June 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Questions 4 and 8 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The equation

$$x^2 + x + 5 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Show that $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = -\dfrac{9}{5}$. *(2 marks)*

(d) Find a quadratic equation, with integer coefficients, which has roots $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$. *(2 marks)*

2 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$3\mathrm{i}z + 2z^*$$

where z^* is the complex conjugate of z. *(3 marks)*

(b) Find the complex number z such that

$$3\mathrm{i}z + 2z^* = 7 + 8\mathrm{i}$$ *(3 marks)*

3 For each of the following improper integrals, find the value of the integral **or** explain briefly why it does not have a value:

(a) $\displaystyle\int_9^\infty \frac{1}{\sqrt{x}}\,\mathrm{d}x$; *(3 marks)*

(b) $\displaystyle\int_9^\infty \frac{1}{x\sqrt{x}}\,\mathrm{d}x$. *(4 marks)*

4 [**Figure 1** and **Figure 2**, printed on the insert, are provided for use in this question.]

The variables x and y are related by an equation of the form

$$y = ax + \frac{b}{x+2}$$

where a and b are constants.

(a) The variables X and Y are defined by $X = x(x+2)$, $Y = y(x+2)$.

Show that $Y = aX + b$. *(2 marks)*

(b) The following approximate values of x and y have been found:

x	1	2	3	4
y	0.40	1.43	2.40	3.35

(i) Complete the table in **Figure 1**, showing values of X and Y. *(2 marks)*

(ii) Draw on **Figure 2** a linear graph relating X and Y. *(2 marks)*

(iii) Estimate the values of a and b. *(3 marks)*

5 (a) Find, in **radians**, the general solution of the equation

$$\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

giving your answer in terms of π. *(5 marks)*

(b) Hence find the smallest **positive** value of x which satisfies this equation. *(2 marks)*

6 The matrices **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

(a) Calculate the matrix **AB**. *(2 marks)*

(b) Show that $\mathbf{A}^2$ is of the form $k\mathbf{I}$, where k is an integer and **I** is the 2×2 identity matrix. *(2 marks)*

(c) Show that $(\mathbf{AB})^2 \neq \mathbf{A}^2\mathbf{B}^2$. *(3 marks)*

Turn over ▶

7 A curve C has equation

$$y = 7 + \frac{1}{x+1}$$

(a) Define the translation which transforms the curve with equation $y = \frac{1}{x}$ onto the curve C. *(2 marks)*

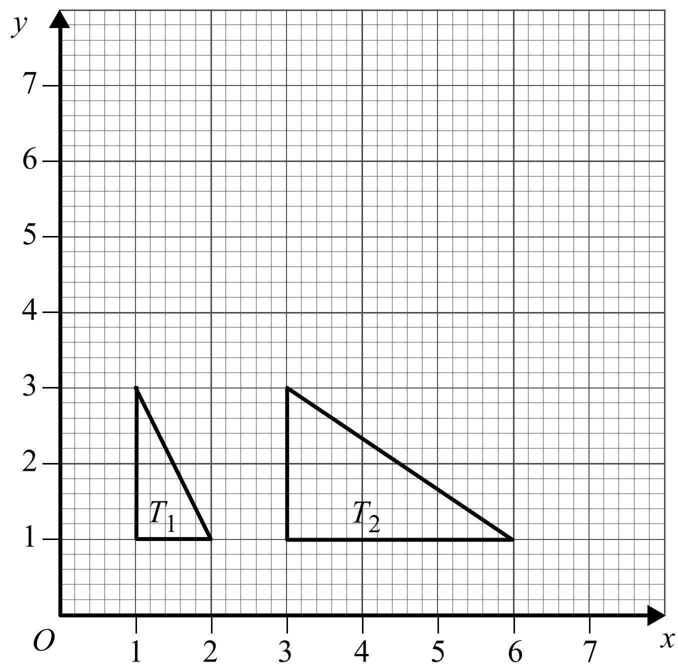
(b) (i) Write down the equations of the two asymptotes of C. *(2 marks)*

(ii) Find the coordinates of the points where the curve C intersects the coordinate axes. *(3 marks)*

(c) Sketch the curve C and its two asymptotes. *(3 marks)*

8 [**Figure 3**, printed on the insert, is provided for use in this question.]

The diagram shows two triangles, T_1 and T_2.



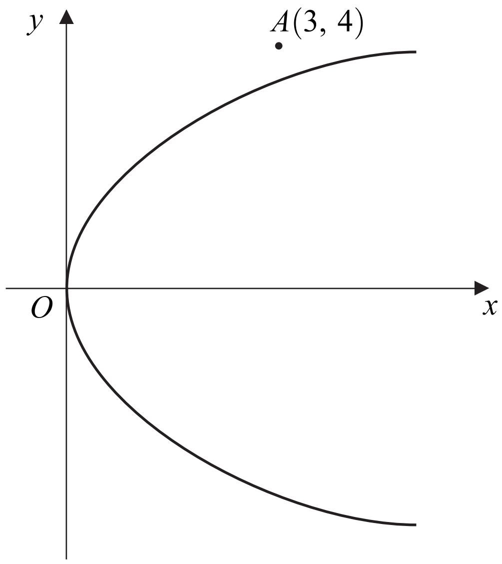
(a) Find the matrix of the stretch which maps T_1 to T_2. *(2 marks)*

(b) The triangle T_2 is reflected in the line $y = x$ to give a third triangle, T_3.

On **Figure 3**, draw the triangle T_3. *(2 marks)*

(c) Find the matrix of the transformation which maps T_1 to T_3. *(3 marks)*

9 The diagram shows the parabola $y^2 = 4x$ and the point A with coordinates $(3, 4)$.



(a) Find an equation of the straight line having gradient m and passing through the point $A(3, 4)$. *(2 marks)*

(b) Show that, if this straight line intersects the parabola, then the y-coordinates of the points of intersection satisfy the equation

$$my^2 - 4y + (16 - 12m) = 0$$ *(3 marks)*

(c) By considering the discriminant of the equation in part (b), find the equations of the two tangents to the parabola which pass through A.

(No credit will be given for solutions based on differentiation.) *(5 marks)*

(d) Find the coordinates of the points at which these tangents touch the parabola. *(4 marks)*

END OF QUESTIONS

Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS **MFP1**
Unit Further Pure 1

ASSESSMENT and QUALIFICATIONS ALLIANCE

Insert

Insert for use in **Questions 4 and 8**.

Fill in the boxes at the top of this page.

Fasten this insert securely to your answer book.

Turn over for Figure 1

Turn over ▶

Figure 1 (for use in Question 4)

x	1	2	3	4
y	0.40	1.43	2.40	3.35
X	3			
Y	1.20			

Figure 2 (for use in Question 4)

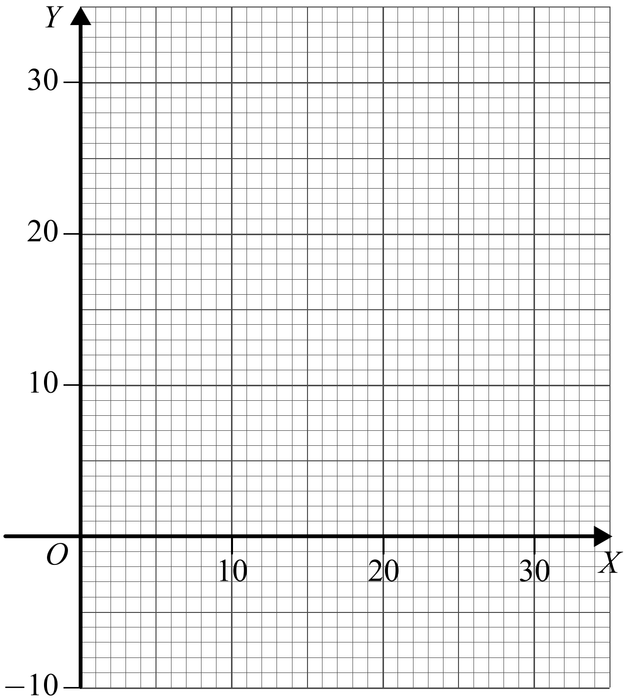
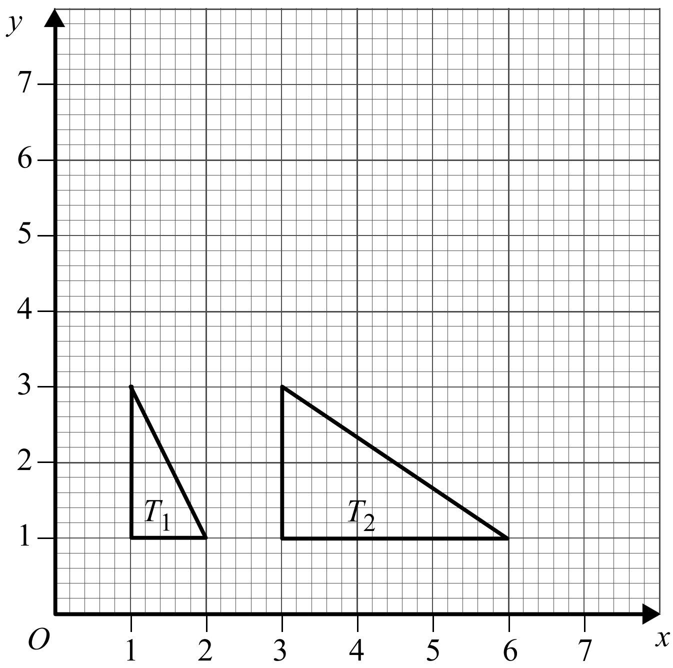


Figure 3 (for use in Question 8)



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Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
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General Certificate of Education
Advanced Subsidiary Examination
June 2009

Mathematics MFP1

Unit Further Pure 1

Specimen paper for examinations in June 2010 onwards

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the space provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The equation

$$2x^2 + x - 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Find the value of $\alpha^2 + \beta^2$. *(2 marks)*

(c) Find a quadratic equation which has roots $4\alpha^2$ and $4\beta^2$. Give your answer in the form $x^2 + px + q = 0$, where p and q are integers. *(3 marks)*

QUESTION PART REFERENCE

2 A curve has equation

$$y = x^2 - 6x + 5$$

The points A and B on the curve have x-coordinates 2 and $2 + h$ respectively.

(a) Find, in terms of h, the gradient of the line AB, giving your answer in its simplest form. *(5 marks)*

(b) Explain how the result of part **(a)** can be used to find the gradient of the curve at A. State the value of this gradient. *(3 marks)*

QUESTION PART REFERENCE

3 The complex number z is defined by

$$z = x + 2\mathrm{i}$$

where x is real.

(a) Find, in terms of x, the real and imaginary parts of:

(i) z^2; *(3 marks)*

(ii) $z^2 + 2z^*$. *(2 marks)*

(b) Show that there is exactly one value of x for which $z^2 + 2z^*$ is real. *(2 marks)*

QUESTION PART REFERENCE

4 The variables x and y are known to be related by an equation of the form

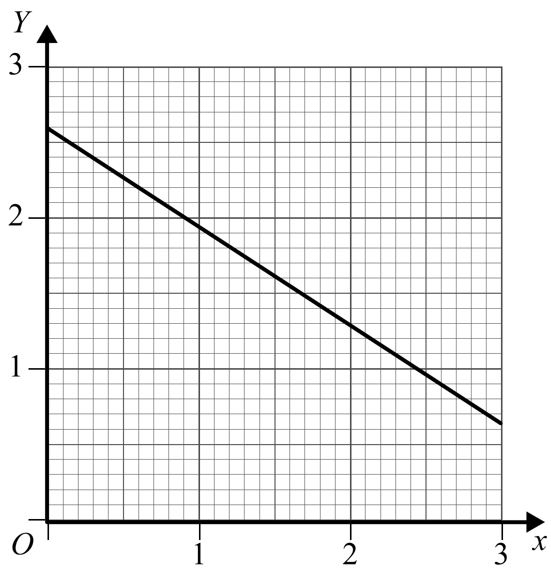
$$y = ab^x$$

where a and b are constants.

(a) Given that $Y = \log_{10} y$, show that x and Y must satisfy an equation of the form

$$Y = mx + c$$ *(3 marks)*

(b) The diagram shows the linear graph which has equation $Y = mx + c$.



Use this graph to calculate:

(i) an approximate value of y when $x = 2.3$, giving your answer to one decimal place;

(ii) an approximate value of x when $y = 80$, giving your answer to one decimal place.

(You are not required to find the values of m and c.) *(4 marks)*

QUESTION PART REFERENCE

5 (a) Find the general solution of the equation

$$\cos(3x - \pi) = \frac{1}{2}$$

giving your answer in terms of π. *(6 marks)*

(b) From your general solution, find all the solutions of the equation which lie between 10π and 11π. *(3 marks)*

QUESTION PART REFERENCE

6 An ellipse E has equation

$$\frac{x^2}{3} + \frac{y^2}{4} = 1$$

(a) Sketch the ellipse E, showing the coordinates of the points of intersection of the ellipse with the coordinate axes. *(3 marks)*

(b) The ellipse E is stretched with scale factor 2 parallel to the y-axis.

Find and simplify the equation of the curve after the stretch. *(3 marks)*

(c) The **original** ellipse, E, is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$. The equation of the translated ellipse is

$$4x^2 + 3y^2 - 8x + 6y = 5$$

Find the values of a and b. *(5 marks)*

QUESTION PART REFERENCE

7 (a) Using surd forms where appropriate, find the matrix which represents:

(i) a rotation about the origin through 30° anticlockwise; *(2 marks)*

(ii) a reflection in the line $y = \dfrac{1}{\sqrt{3}}x$. *(2 marks)*

(b) The matrix **A**, where

$$\mathbf{A} = \begin{bmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

represents a combination of an enlargement and a reflection. Find the scale factor of the enlargement and the equation of the mirror line of the reflection. *(2 marks)*

(c) The transformation represented by **A** is followed by the transformation represented by **B**, where

$$\mathbf{B} = \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$$

Find the matrix of the combined transformation and give a full geometrical description of this combined transformation. *(5 marks)*

QUESTION PART REFERENCE

8 A curve has equation

$$y = \frac{x^2}{(x-1)(x-5)}$$

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) Show that the curve has no point of intersection with the line $y = -1$. *(3 marks)*

(c) (i) Show that, if the curve intersects the line $y = k$, then the x-coordinates of the points of intersection must satisfy the equation

$$(k-1)x^2 - 6kx + 5k = 0$$ *(2 marks)*

(ii) Show that, if this equation has equal roots, then

$$k(4k+5) = 0$$ *(2 marks)*

(d) Hence find the coordinates of the two stationary points on the curve. *(5 marks)*

QUESTION PART REFERENCE

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
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General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MFP1

Unit Further Pure 1

Thursday 27 May 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 A curve passes through the point (1, 3) and satisfies the differential equation

$$\frac{dy}{dx} = 1 + x^3$$

Starting at the point (1, 3), use a step-by-step method with a step length of 0.1 to estimate the y-coordinate of the point on the curve for which $x = 1.3$. Give your answer to three decimal places.

(No credit will be given for methods involving integration.) *(6 marks)*

QUESTION PART REFERENCE

2 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(1 - 2\mathrm{i})z - z^*$$

(4 marks)

(b) Hence find the complex number z such that

$$(1 - 2\mathrm{i})z - z^* = 10(2 + \mathrm{i})$$

(2 marks)

QUESTION PART REFERENCE

3 Find the general solution, in degrees, of the equation

$$\cos(5x - 20°) = \cos 40°$$

(5 marks)

QUESTION PART REFERENCE

4 The variables x and y are related by an equation of the form

$$y = ax^2 + b$$

where a and b are constants.

The following approximate values of x and y have been found.

x	2	4	6	8
y	6.0	10.5	18.0	28.2

(a) Complete the table below, showing values of X, where $X = x^2$. *(1 mark)*

(b) On the diagram below, draw a linear graph relating X and y. *(2 marks)*

(c) Use your graph to find estimates, to two significant figures, for:

(i) the value of x when $y = 15$; *(2 marks)*

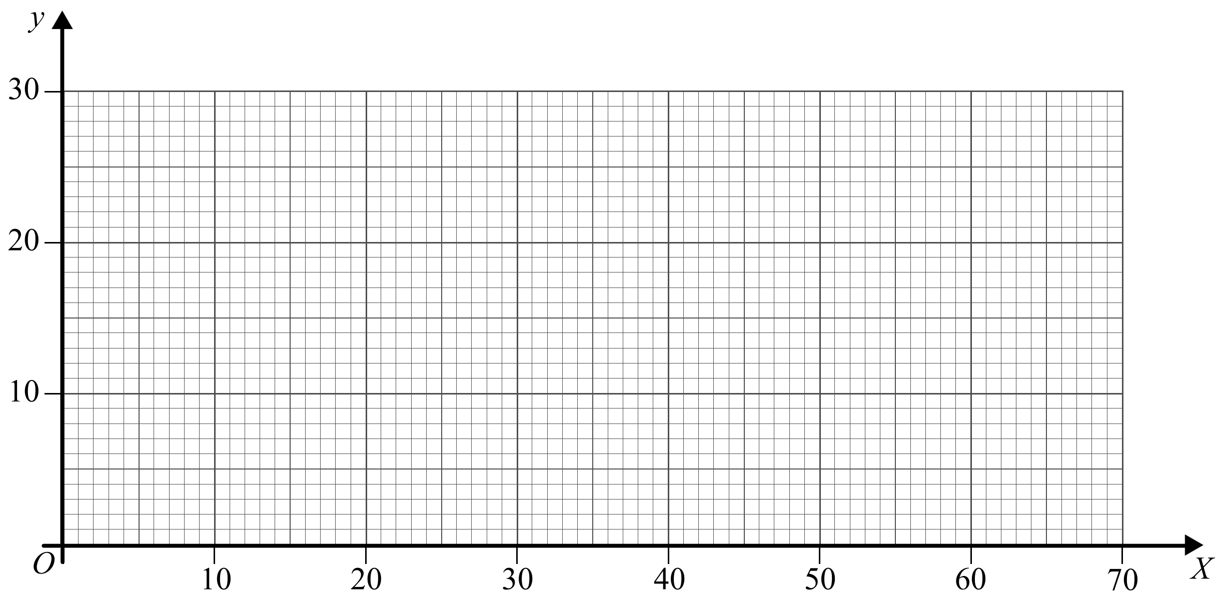
(ii) the values of a and b. *(3 marks)*

QUESTION PART REFERENCE

(a)

x	2	4	6	8
X				
y	6.0	10.5	18.0	28.2

(b)



5 A curve has equation $y = x^3 - 12x$.

The point A on the curve has coordinates $(2, -16)$.

The point B on the curve has x-coordinate $2 + h$.

(a) Show that the gradient of the line AB is $6h + h^2$. *(4 marks)*

(b) Explain how the result of part **(a)** can be used to show that A is a stationary point on the curve. *(2 marks)*

QUESTION PART REFERENCE

6 The matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Describe fully the geometrical transformation represented by each of the following matrices:

(a) $\mathbf{A}$; *(2 marks)*

(b) $\mathbf{B}$; *(2 marks)*

(c) $\mathbf{A}^2$; *(2 marks)*

(d) $\mathbf{B}^2$; *(2 marks)*

(e) $\mathbf{AB}$. *(3 marks)*

QUESTION PART REFERENCE

7 (a) (i) Write down the equations of the two asymptotes of the curve $y = \frac{1}{x-3}$. *(2 marks)*

(ii) Sketch the curve $y = \frac{1}{x-3}$, showing the coordinates of any points of intersection with the coordinate axes. *(2 marks)*

(iii) On the same axes, again showing the coordinates of any points of intersection with the coordinate axes, sketch the line $y = 2x - 5$. *(1 mark)*

(b) (i) Solve the equation

$$\frac{1}{x-3} = 2x - 5$$ *(3 marks)*

(ii) Find the solution of the inequality

$$\frac{1}{x-3} < 2x - 5$$ *(2 marks)*

QUESTION PART REFERENCE

8 The quadratic equation

$$x^2 - 4x + 10 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{2}{5}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots $\alpha + \frac{2}{\beta}$ and $\beta + \frac{2}{\alpha}$. *(6 marks)*

QUESTION PART REFERENCE

9 A parabola P has equation $y^2 = x - 2$.

(a) (i) Sketch the parabola P. *(2 marks)*

(ii) On your sketch, draw the two tangents to P which pass through the point $(-2, 0)$. *(2 marks)*

(b) (i) Show that, if the line $y = m(x + 2)$ intersects P, then the x-coordinates of the points of intersection must satisfy the equation

$$m^2x^2 + (4m^2 - 1)x + (4m^2 + 2) = 0$$ *(3 marks)*

(ii) Show that, if this equation has equal roots, then

$$16m^2 = 1$$ *(3 marks)*

(iii) Hence find the coordinates of the points at which the tangents to P from the point $(-2, 0)$ touch the parabola P. *(3 marks)*

QUESTION PART REFERENCE

General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MFP1

Unit Further Pure 1

Friday 20 May 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 A curve passes through the point (2, 3) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{\sqrt{2+x}}$$

Starting at the point (2, 3), use a step-by-step method with a step length of 0.5 to estimate the value of y at $x = 3$. Give your answer to four decimal places.

(5 marks)

---

2 The equation

$$4x^2 + 6x + 3 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = \frac{3}{4}$. *(2 marks)*

(c) Find an equation, with integer coefficients, which has roots

$$3\alpha - \beta \quad \text{and} \quad 3\beta - \alpha$$

(5 marks)

---

3 It is given that $z = x + iy$, where x and y are real.

(a) Find, in terms of x and y, the real and imaginary parts of

$$(z - i)(z^* - i)$$

(3 marks)

(b) Given that

$$(z - i)(z^* - i) = 24 - 8i$$

find the two possible values of z. *(4 marks)*

4 The variables x and Y, where $Y = \log_{10} y$, are related by the equation

$$Y = mx + c$$

where m and c are constants.

(a) Given that $y = ab^x$, express a in terms of c, and b in terms of m. *(3 marks)*

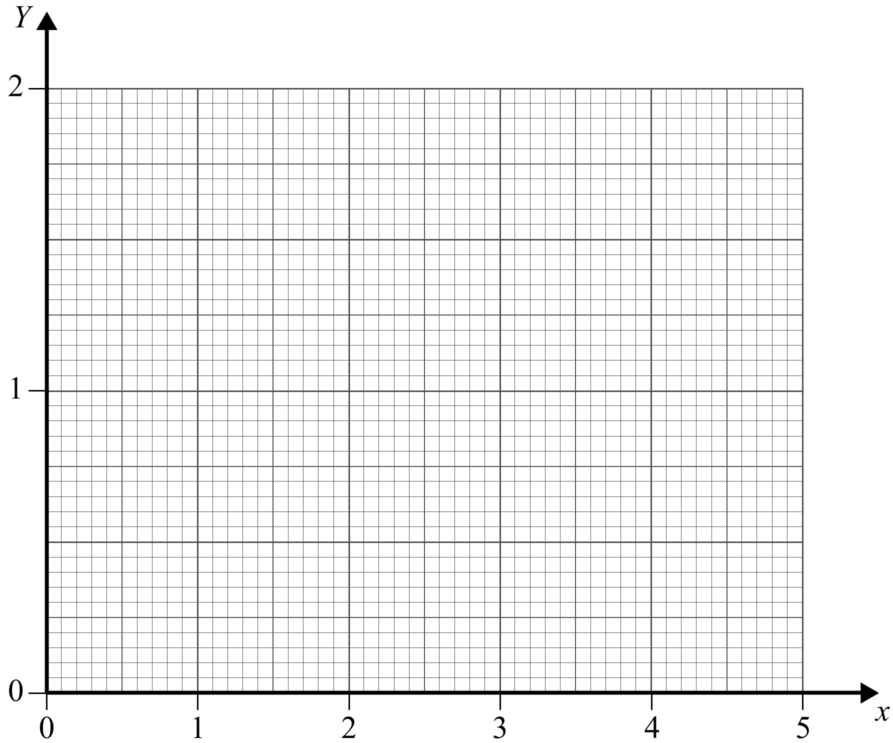
(b) It is given that $y = 12$ when $x = 1$ and that $y = 27$ when $x = 5$.

On the diagram below, draw a linear graph relating x and Y. *(3 marks)*

(c) Use your graph to estimate, to two significant figures:

(i) the value of y when $x = 3$; *(2 marks)*

(ii) the value of a. *(2 marks)*



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5 (a) Find the general solution of the equation

$$\cos\left(3x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

giving your answer in terms of π. *(5 marks)*

(b) Use your general solution to find the smallest solution of this equation which is greater than 5π. *(2 marks)*

6 (a) Expand $(5+h)^3$. *(1 mark)*

(b) A curve has equation $y = x^3 - x^2$.

(i) Find the gradient of the line passing through the point $(5, 100)$ and the point on the curve for which $x = 5 + h$. Give your answer in the form

$$p + qh + rh^2$$

where p, q and r are integers. *(4 marks)*

(ii) Show how the answer to part **(b)(i)** can be used to find the gradient of the curve at the point $(5, 100)$. State the value of this gradient. *(2 marks)*

---

7 The matrix $\mathbf{A}$ is defined by

$$\mathbf{A} = \begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$$

(a) (i) Calculate the matrix $\mathbf{A}^2$. *(2 marks)*

(ii) Show that $\mathbf{A}^3 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(b) Describe the single geometrical transformation, or combination of two geometrical transformations, corresponding to each of the matrices:

(i) $\mathbf{A}^3$; *(2 marks)*

(ii) $\mathbf{A}$. *(3 marks)*

---

8 A curve has equation $y = \dfrac{1}{x^2 - 4}$.

(a) (i) Write down the equations of the three asymptotes of the curve. *(3 marks)*

(ii) Sketch the curve, showing the coordinates of any points of intersection with the coordinate axes. *(4 marks)*

(b) Hence, or otherwise, solve the inequality

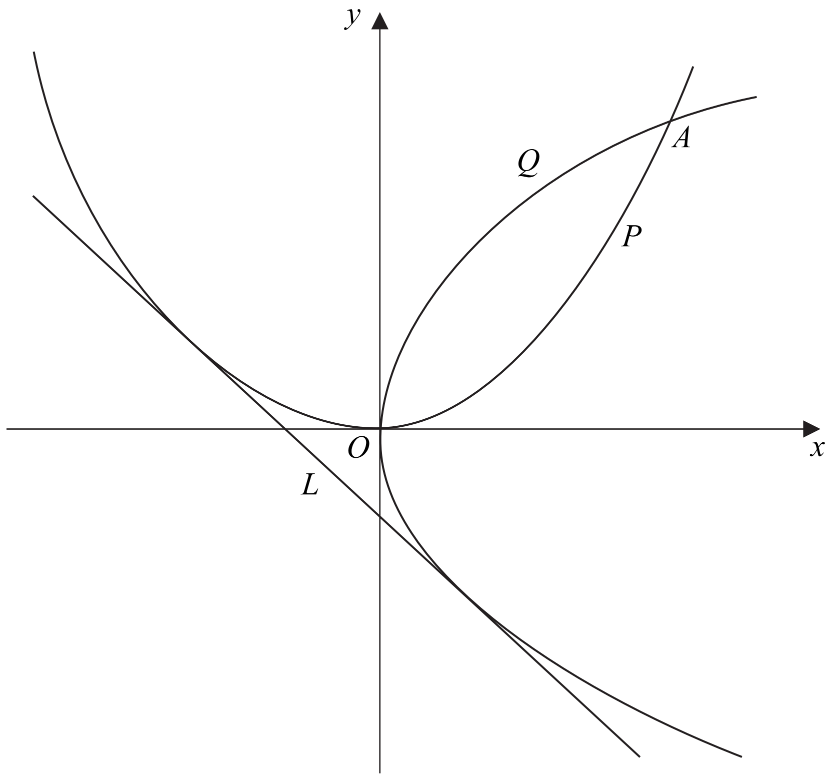
$$\frac{1}{x^2 - 4} < -2$$

(3 marks)

9 The diagram shows a parabola P which has equation $y = \frac{1}{8}x^2$, and another parabola Q which is the image of P under a reflection in the line $y = x$.

The parabolas P and Q intersect at the origin and again at a point A.

The line L is a tangent to both P and Q.



(a) (i) Find the coordinates of the point A. *(2 marks)*

(ii) Write down an equation for Q. *(1 mark)*

(iii) Give a reason why the gradient of L must be -1. *(1 mark)*

(b) (i) Given that the line $y = -x + c$ intersects the parabola P at two distinct points, show that

$$c > -2$$ *(3 marks)*

(ii) Find the coordinates of the points at which the line L touches the parabolas P and Q.

(No credit will be given for solutions based on differentiation.) *(4 marks)*

END OF QUESTIONS

General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MFP1

Unit Further Pure 1

Friday 18 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The quadratic equation

$$5x^2 - 7x + 1 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = \dfrac{39}{5}$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\alpha + \frac{1}{\alpha} \quad \text{and} \quad \beta + \frac{1}{\beta}$$ *(5 marks)*

---

2 A curve has equation $y = x^4 + x$.

(a) Find the gradient of the line passing through the point $(-2, 14)$ and the point on the curve for which $x = -2 + h$. Give your answer in the form

$$p + qh + rh^2 + h^3$$

where p, q and r are integers. *(5 marks)*

(b) Show how the answer to part **(a)** can be used to find the gradient of the curve at the point $(-2, 14)$. State the value of this gradient. *(2 marks)*

---

3 It is given that $z = x + \mathrm{i}y$, where x and y are real numbers.

(a) Find, in terms of x and y, the real and imaginary parts of

$$\mathrm{i}(z + 7) + 3(z^* - \mathrm{i})$$ *(3 marks)*

(b) Hence find the complex number z such that

$$\mathrm{i}(z + 7) + 3(z^* - \mathrm{i}) = 0$$ *(3 marks)*

---

4 Find the general solution, in degrees, of the equation

$$\sin\left(70° - \tfrac{2}{3}x\right) = \cos 20°$$ *(6 marks)*

5 The curve C has equation $y = \dfrac{x}{(x+1)(x-2)}$.

The line L has equation $y = -\frac{1}{2}$.

(a) Write down the equations of the asymptotes of C. *(3 marks)*

(b) The line L intersects the curve C at two points. Find the x-coordinates of these two points. *(2 marks)*

(c) Sketch C and L on the same axes.

(You are given that the curve C has no stationary points.) *(3 marks)*

(d) Solve the inequality

$$\frac{x}{(x+1)(x-2)} \leqslant -\frac{1}{2}$$ *(3 marks)*

---

6 (a) Using surd forms, find the matrix of a rotation about the origin through 135° anticlockwise. *(2 marks)*

(b) The matrix $\mathbf{M}$ is defined by $\mathbf{M} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$.

(i) Given that $\mathbf{M}$ represents an enlargement followed by a rotation, find the scale factor of the enlargement and the angle of the rotation. *(3 marks)*

(ii) The matrix $\mathbf{M}^2$ also represents an enlargement followed by a rotation. State the scale factor of the enlargement and the angle of the rotation. *(2 marks)*

(iii) Show that $\mathbf{M}^4 = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix. *(2 marks)*

(iv) Deduce that $\mathbf{M}^{2012} = -2^n\mathbf{I}$ for some positive integer n. *(2 marks)*

Turn over ▶

7 The equation

$$24x^3 + 36x^2 + 18x - 5 = 0$$

has one real root, α.

(a) Show that α lies in the interval $0.1 < x < 0.2$. *(2 marks)*

(b) Starting from the interval $0.1 < x < 0.2$, use interval bisection **twice** to obtain an interval of width 0.025 within which α must lie. *(3 marks)*

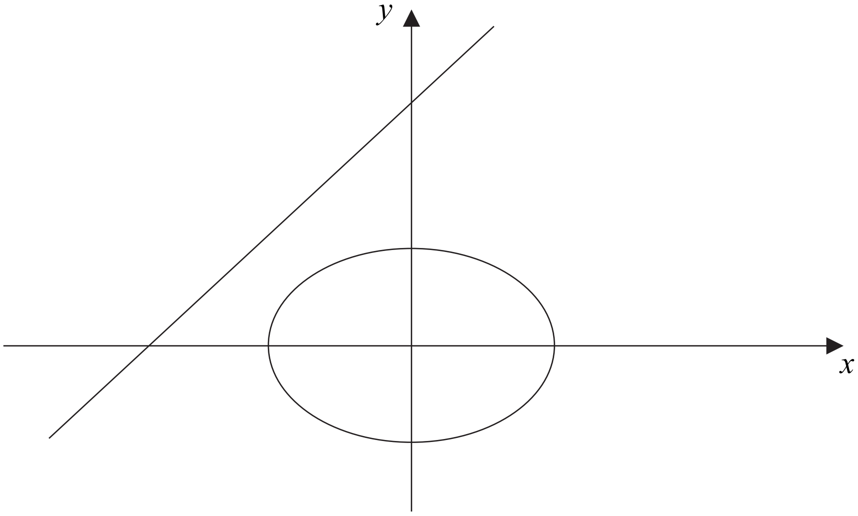
(c) Taking $x_1 = 0.2$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α. Give your answer to four decimal places. *(4 marks)*

8 The diagram shows the ellipse E with equation

$$\frac{x^2}{5} + \frac{y^2}{4} = 1$$

and the straight line L with equation

$$y = x + 4$$



(a) Write down the coordinates of the points where the ellipse E intersects the coordinate axes. *(2 marks)*

(b) The ellipse E is translated by the vector $\begin{bmatrix} p \\ 0 \end{bmatrix}$, where p is a constant. Write down the equation of the translated ellipse. *(2 marks)*

(c) Show that, if the translated ellipse intersects the line L, the x-coordinates of the points of intersection must satisfy the equation

$$9x^2 - (8p - 40)x + (4p^2 + 60) = 0$$ *(3 marks)*

(d) Given that the line L is a tangent to the translated ellipse, find the coordinates of the two possible points of contact.

(No credit will be given for solutions based on differentiation.) *(8 marks)*

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
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General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MFP1

Unit Further Pure 1

Friday 17 May 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 The equation

$$x^3 - x^2 + 4x - 900 = 0$$

has exactly one real root, α.

Taking $x_1 = 10$ as a first approximation to α, use the Newton–Raphson method to find a second approximation, x_2, to α. Give your answer to four significant figures.

(3 marks)

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

2 The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined by

$$\mathbf{A} = \begin{bmatrix} p & 2 \\ 4 & p \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 & 1 \\ 2 & 3 \end{bmatrix}$$

(a) Find, in terms of p, the matrices:

(i) $\mathbf{A} - \mathbf{B}$; *(1 mark)*

(ii) $\mathbf{AB}$. *(2 marks)*

(b) Show that there is a value of p for which $\mathbf{A} - \mathbf{B} + \mathbf{AB} = k\mathbf{I}$, where k is an integer and $\mathbf{I}$ is the 2×2 identity matrix, and state the corresponding value of k. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 2

3 (a) Find the general solution, in degrees, of the equation

$$\cos(5x + 40°) = \cos 65°$$

(5 marks)

(b) Given that

$$\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

express $\sin\frac{\pi}{12}$ in the form $\left(\cos\frac{\pi}{4}\right)\left(\cos(a\pi) + \cos(b\pi)\right)$, where a and b are rational.

(3 marks)

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

4 (a) It is given that $z = x + y\mathrm{i}$, where x and y are real numbers.

(i) Write down, in terms of x and y, an expression for $(z - 2\mathrm{i})^*$. *(1 mark)*

(ii) Solve the equation

$$(z - 2\mathrm{i})^* = 4\mathrm{i}z + 3$$

giving your answer in the form $a + b\mathrm{i}$. *(5 marks)*

(b) It is given that $p + q\mathrm{i}$, where p and q are real numbers, is a root of the equation $z^2 + 10\mathrm{i}z - 29 = 0$.

Without finding the values of p and q, **state** why $p - q\mathrm{i}$ is **not** a root of the equation $z^2 + 10\mathrm{i}z - 29 = 0$. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

5 (a) A curve has equation $y = 2x^2 - 5x$.

The point P on the curve has coordinates $(1, -3)$.

The point Q on the curve has x-coordinate $1 + h$.

(i) Show that the gradient of the line PQ is $2h - 1$. *(3 marks)*

(ii) Explain how the result of part **(a)(i)** can be used to show that the tangent to the curve at the point P is parallel to the line $x + y = 0$. *(2 marks)*

(b) For the improper integral $\int_1^{\infty} x^{-4}(2x^2 - 5x)\,\mathrm{d}x$, either show that the integral has a finite value and state its value, or explain why the integral does not have a finite value. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 5

6 The equation

$$2x^2 + 3x - 6 = 0$$

has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$. *(2 marks)*

(b) Hence show that $\alpha^3 + \beta^3 = -\frac{135}{8}$. *(3 marks)*

(c) Find a quadratic equation, with integer coefficients, whose roots are $\alpha + \frac{\alpha}{\beta^2}$ and $\beta + \frac{\beta}{\alpha^2}$. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

7 (a) Show that the equation $4x^3 - x - 540\,000 = 0$ has a root, α, in the interval $51 < \alpha < 52$. *(2 marks)*

(b) It is given that $S_n = \sum_{r=1}^{n} (2r-1)^2$.

(i) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that $S_n = \frac{n}{3}(kn^2 - 1)$, where k is an integer to be found. *(5 marks)*

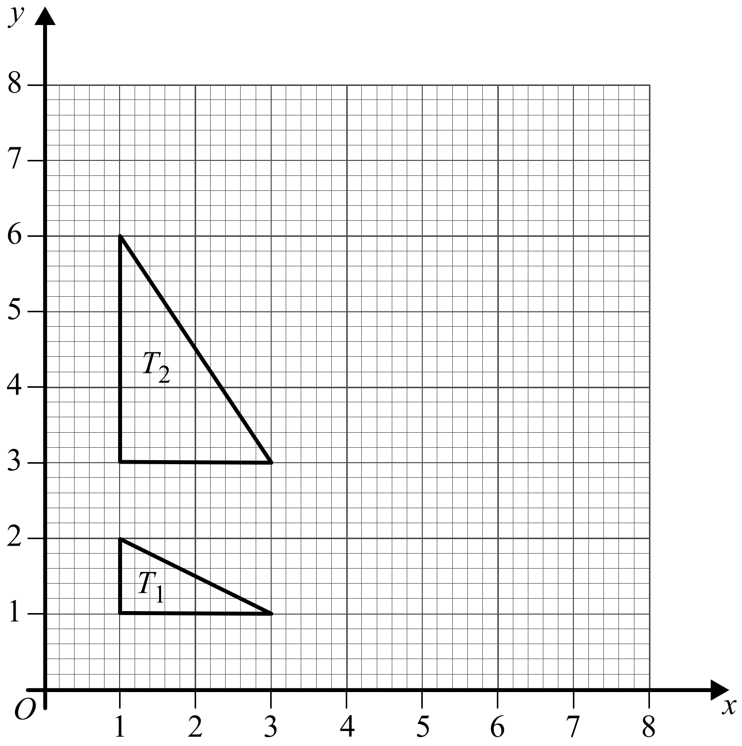
(ii) Hence show that $6S_n$ can be written as the product of three consecutive integers. *(2 marks)*

(c) Find the smallest value of N for which the sum of the squares of the first N odd numbers is greater than $180\,000$. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 7

8 The diagram shows two triangles, T_1 and T_2.



(a) Find the matrix which represents the stretch that maps triangle T_1 onto triangle T_2. *(2 marks)*

(b) The triangle T_2 is reflected in the line $y = \sqrt{3}x$ to give a third triangle, T_3. Find, using surd forms where appropriate:

(i) the matrix which represents the reflection that maps triangle T_2 onto triangle T_3; *(2 marks)*

(ii) the matrix which represents the combined transformation that maps triangle T_1 onto triangle T_3. *(2 marks)*

QUESTION PART REFERENCE	**Answer space for question 8**

9 A curve has equation

$$y = \frac{x^2 - 2x + 1}{x^2 - 2x - 3}$$

(a) Find the equations of the three asymptotes of the curve. *(3 marks)*

(b) (i) Show that if the line $y = k$ intersects the curve then

$$(k-1)x^2 - 2(k-1)x - (3k+1) = 0$$ *(1 mark)*

(ii) Given that the equation $(k-1)x^2 - 2(k-1)x - (3k+1) = 0$ has real roots, show that

$$k^2 - k \geqslant 0$$ *(3 marks)*

(iii) Hence show that the curve has only one stationary point and find its coordinates.

(No credit will be given for solutions based on differentiation.) *(4 marks)*

(c) Sketch the curve and its asymptotes. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 9

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
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AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2014

Mathematics MFP1

Unit Further Pure 1

Tuesday 10 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 A curve passes through the point $(9,\ 6)$ and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2+\sqrt{x}}$$

Use a step-by-step method with a step length of 0.25 to estimate the value of y at $x = 9.5$. Give your answer to four decimal places.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 1

2 The quadratic equation

$$2x^2 + 8x + 1 = 0$$

has roots α and β.

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

[2 marks]

(b) (i) Find the value of $\alpha^2 + \beta^2$.

[2 marks]

(ii) Hence, or otherwise, show that $\alpha^4 + \beta^4 = \dfrac{449}{2}$.

[2 marks]

(c) Find a quadratic equation, with integer coefficients, which has roots

$$2\alpha^4 + \frac{1}{\beta^2} \text{ and } 2\beta^4 + \frac{1}{\alpha^2}$$

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

3 Use the formulae for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to find the value of

$$\sum_{r=3}^{60} r^2(r-6)$$

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

4 Find the complex number z such that

$$5\mathrm{i}z + 3z^* + 16 = 8\mathrm{i}$$

Give your answer in the form $a + b\mathrm{i}$, where a and b are real.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 4

5 A curve C has equation $y = x(x+3)$.

(a) Find the gradient of the line passing through the point $(-5,\ 10)$ and the point on C with x-coordinate $-5+h$. Give your answer in its simplest form.

[3 marks]

(b) Show how the answer to part **(a)** can be used to find the gradient of the curve C at the point $(-5,\ 10)$. State the value of this gradient.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 5

6 A curve C has equation $y = \frac{1}{x(x+2)}$.

(a) Write down the equations of all the asymptotes of C.

[2 marks]

(b) The curve C has exactly one stationary point. The x-coordinate of the stationary point is -1.

(i) Find the y-coordinate of the stationary point.

[1 mark]

(ii) Sketch the curve C.

[2 marks]

(c) Solve the inequality

$$\frac{1}{x(x+2)} \leqslant \frac{1}{8}$$

[5 marks]

QUESTION PART REFERENCE

Answer space for question 6

7 (a) Write down the 2×2 matrix corresponding to each of the following transformations:

(i) a reflection in the line $y = -x$;

[1 mark]

(ii) a stretch parallel to the y-axis of scale factor 7.

[1 mark]

(b) Hence find the matrix corresponding to the combined transformation of a reflection in the line $y = -x$ followed by a stretch parallel to the y-axis of scale factor 7.

[2 marks]

(c) The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} -3 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{bmatrix}$.

(i) Show that $\mathbf{A}^2 = k\mathbf{I}$, where k is a constant and $\mathbf{I}$ is the 2×2 identity matrix.

[1 mark]

(ii) Show that the matrix $\mathbf{A}$ corresponds to a combination of an enlargement and a reflection. State the scale factor of the enlargement and state the equation of the line of reflection in the form $y = (\tan\theta)x$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 7

8 (a) Find the general solution of the equation

$$\cos\left(\frac{5}{4}x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$$

giving your answer for x in terms of π.

[5 marks]

(b) Use your general solution to find the **sum** of all the solutions of the equation $\cos\left(\frac{5}{4}x - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$ that lie in the interval $0 \leqslant x \leqslant 20\pi$. Give your answer in the form $k\pi$, stating the exact value of k.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8

9 An ellipse E has equation

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

(a) Sketch the ellipse E, showing the values of the intercepts on the coordinate axes.

[2 marks]

(b) Given that the line with equation $y = x + k$ intersects the ellipse E at two distinct points, show that $-5 < k < 5$.

[5 marks]

(c) The ellipse E is translated by the vector $\begin{bmatrix} a \\ b \end{bmatrix}$ to form another ellipse whose equation is $9x^2 + 16y^2 + 18x - 64y = c$. Find the values of the constants a, b and c.

[5 marks]

(d) **Hence** find an equation for each of the two tangents to the ellipse $9x^2 + 16y^2 + 18x - 64y = c$ that are parallel to the line $y = x$.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 9**